Chapter 6: Gauss's Law

This Review Guide will cover Gauss's Law, it's relation to flux, and its application to different objects.

Equation:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

in words: The flux of the electric field through any closed surface (i.e. Gaussian Surface) is equal to the net enclosed charge divided by the permeability of free space."

So...what is flux?

- a measure of electric field passing through an area

Useful Relationships for Q_{in} internal analysis:

$\lambda = \frac{Q}{L}$ $\sigma = \frac{Q}{A}$

$\rho = \frac{Q}{V}$

Solving Strategy:

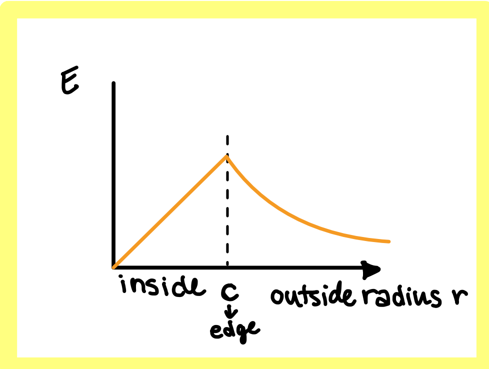
1) draw a proper Gaussian surface for the situation (sphere, cylinder, or prism)

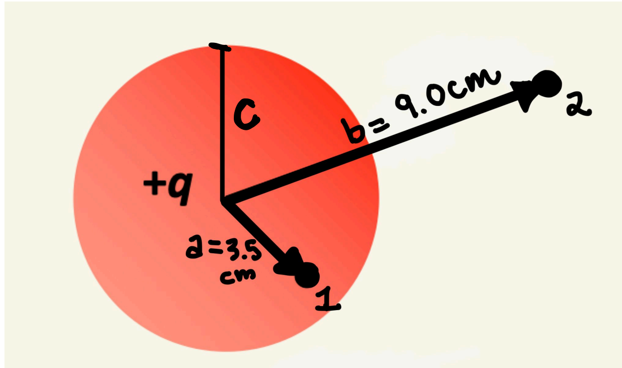
2) Determine Q internal
   a. if radius r ≥ radius of object R, use charge given
   b. if r < R, use relationships above & find the fraction of the whole charge used.

3) Plug in values & solve!

written by: lilie henry

These practice problems will focus on finding the electric field in various charge obejcts while also exploring idea of flux and charge density relationships.





An insulating sphere has a radius 5 cm has a uniform density of ρ and a total charge of 30e-7 C. Calculate the magnitude of the electric field from points a and b respectively.

<table>
<tr><th><u>Work:</u></th><th><u>Explanation:</u></th></tr>
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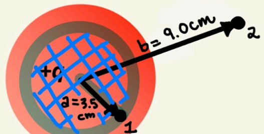
<u>know:</u>
r = 5cm → 0.05 m
Q = 30e-7



1 (point a)

$\oint E\,dA = \frac{q_{in}}{\varepsilon_0}$

$\rho = \frac{Q_{total}}{V_{total}} = \frac{q_{in}}{V_{in}}$

$q_{in} = \frac{Q_{total}V_{in}}{V_{total}}$

$E\oint dA = 4\pi k q_{in}$

$= \frac{(30e-7)(\cancel{\pi}(0.035)^2)}{(\cancel{\pi}(0.05)^2)}$

$E(\cancel{4\pi}r^2) = \cancel{4\pi}k\,q_{in}$

$q_{in} = 1.47e-6$

$E = \frac{kq_{in}}{r^2} = \frac{(9.0e9)(1.47e-6)}{(0.05)^2} = \boxed{5.29e6\ N/C}$

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Starting with Gauss' Law, we are able to draw a spherical Gaussian surface that has an area of $4\pi r^2$. Because **(a)** is less than the total, we have to find the fraction of the charge of the insulating sphere by comparing the total Q/V relationship to the Q_{in}/V_{in} of our Gaussian surface. After setting up the ration and solving for Q internal, we are then able to re-write the Vacuum permittivity constant to cancel out 4 and pi, getting the equation kq_{in}/r^2.

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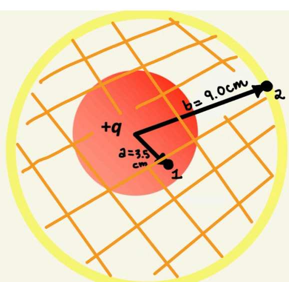

$\oint E\cdot dA = \frac{q_{in}}{\varepsilon_0}$

$E\oint dA = \frac{q_{in}}{\varepsilon_0}$

$E\,(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$

$E = \frac{q_{in}}{\varepsilon_0 4\pi r^2} = \frac{(30e-7)}{(8.85e-12)4\pi(0.09)^2} = \boxed{3.33e6\ N/C}$

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This time, **(b)** is outside of the insulating sphere, meaning that q_{in} is the same as the total charge of the sphere. Using the same type of Gaussian sphere with the area of $4\pi r^2$, we are able to isolate **E** by dividing out the area to the other side and plugging in the given values.

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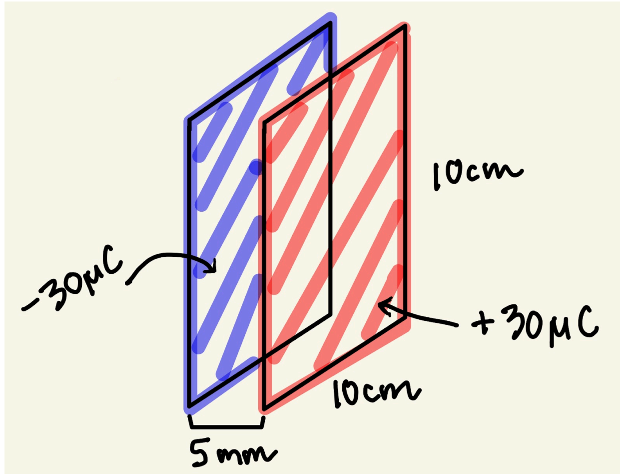
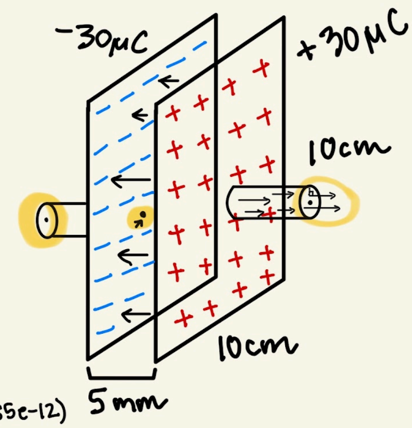
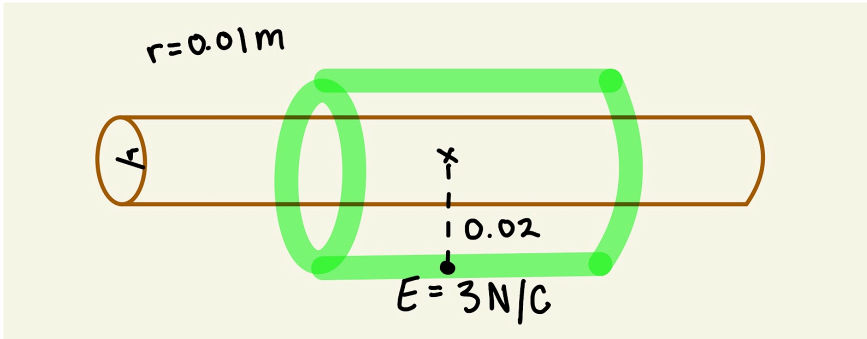
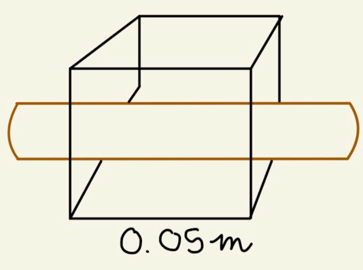


(#81 on Openstax Textbook): Two 10 cm x 10 cm pieces of aluminum foil of thickness 0.1 mm face each other with a separation of 5 mm. One of the foils has a charge of +30 μC and the other has -30 μC. (a) Find the charge density at all surfaces, i.e. on those facing each other and those facing away. (b) Find the electric field between the plates near the center assuming planar symmetry.

Work:	***Explanation:***
a) $\sigma = \frac{Q}{A} = \frac{(30e\text{-}6\ C)}{(0.1 \cdot 0.1)} = \boxed{0.003\ C/m^2}$ (+ plate) $\frac{(-30e\text{-}6\ C)}{(0.1 \cdot 0.1)} = \boxed{-0.003\ C/m^2}$ (− plate)	Charge density is defined as the charge Q over the area A. Using the given charges for each plate, you can plug in the corresponding charge and find the volume by multiplying the dimensions of the plate together.
b) (one plate) $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ $E \oint d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ $E(2A) = \frac{q_{in}}{\varepsilon_0}$ $E = \frac{q_{in}}{2A\varepsilon_0} = \frac{(30e\text{-}6)}{2(0.1 \cdot 0.1)(8.85e\text{-}12)}$ $= 1.69e8\ N/C \cdot 2$ $= \boxed{3.38e8\ N/C}$  	To find the electric field, we once again begin with Gauss' Law. For the area, we are using cylinders, but only considering the "**cap" because there are no perpendicular field lines passing through walls of the cylinder**. Because there are 2 caps, one on each side of the plate, the area (defined as A for now), is multiplied by two. Then, E is isolated and the values for q_{in}, A, and the vacuum permeability constant is plugged in. However, this equation only accounts for one of the plates, so the answer needs to be **multiplied by two**.



(#51 on OpenStax): The electric field at 2cm from the center of a long copper rod of radius 1 cm has a magnitude 3 N/C and directed outward from the axis of the rod. (a) How much charge per unit length exists on the copper rod? (b) What would be the electric flux through a cube of side 5 cm situated such that the rod passes through opposite sides of the cube perpendicularly?

Work:	***Explanation:***
a) $\lambda = \frac{Q}{L}$ $\oint E\, dA = \frac{q_{in}}{\varepsilon_0}$ $E(\cancel{2}\cancel{\pi} rL) = \cancel{4}^{2}\cancel{\pi} k\, q_{in}$ $ErL = 2k\, q_{in}$ $\frac{q_{in}}{L} = \lambda$ $\lambda = \frac{Er}{2k} = \frac{(3)(0.02)}{2(9.0e9)} = \boxed{3.3e\text{-}12\ C/m}$	Using a similar process to the previous problem, we can solve for λ (Charge Q over Length L) by using a cylindrical Gaussian surface but we wil **only consider the body of the cylinder because the field lines only pass through the walls**. By using the given radius for the Eletric Field, we can solve for it.
b) $\Phi_E = \frac{q_{in}}{\varepsilon_0}$ $\Phi_E = \frac{1.65e\text{-}13\,C}{8.85e\text{-}12}$ $\boxed{\Phi_E = 0.019\ N\cdot m^2/C}$ $\lambda = \frac{Q}{L}$ $Q_{in} = \lambda L = (3.3e\text{-}12)(0.05)$ $= 1.65e\text{-}13\,C$ 	**Electric flux is equal to the internal charge over the vacuum permeability constant**. We can use the relationship of our newly found λ isolate Q internal, using the length of the cube as the L value. Once we have the value for the internal charge, we can then plug it back into the flux equation and divide it by the vacuum permeability constant.