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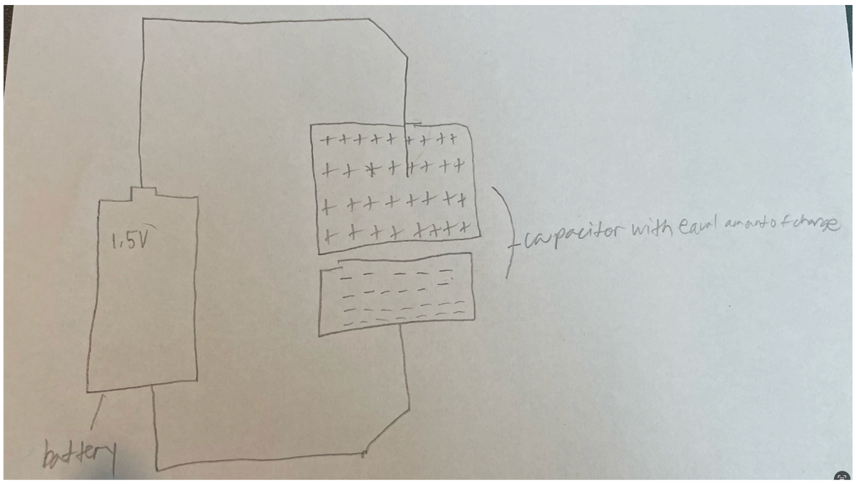
Chapter 8: Capacitance
AP Review Guide

Background:

In chapter 8, we will look at capacitors and what factors give them a certain capacitance, how to calculate this capacitance in different capacitors, charging a capacitor, capacitors in a loop, how to calculate the energy in a capacitor, and dielectrics.

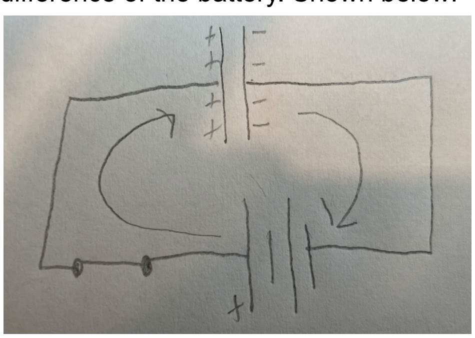
Important terms:

1. Capacitance: Conductor's ability to store charge; occurs when a battery creates a potential difference and pushes positive and negative charge on conductors.

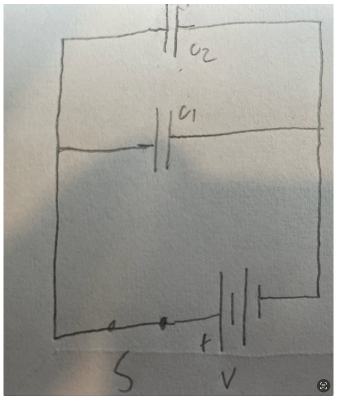


(Capacitors have equal amounts of charge)

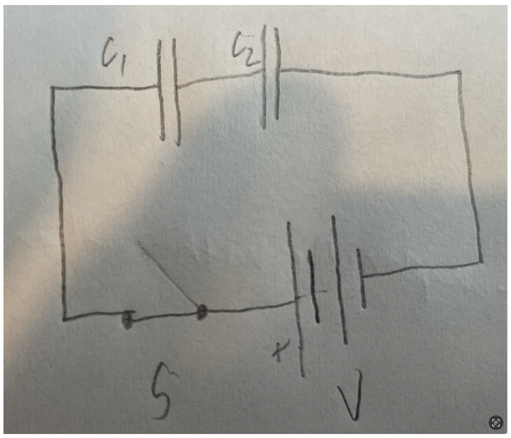
2. Types of Capacitors: Parallel plates, Cylindrical Capacitor (Wire surrounded by a cylindrical conducting shell), Spherical capacitor (conducting sphere surrounded by a conducting sphere)
3. Charging: When the battery switch is closed, positive charge move from the positive end of the battery to one side of the capacitor leaving negative charges on the other side. This current continues until there is no charge left for the battery to push, and the potential difference on the capacitor reaches the potential difference of the battery. Shown below:



4. Capacitors in parallel: For capacitors in parallel, Vbattery = V1 = V2



5. Capacitors in Series: For capacitors in series, Qbattery = Q1 = Q2



7. Energy in a Capacitor: How much work is needed to store a specific charge on a capacitor.
8. Dielectric: An insulator between the the conducting surfaces of a capacitor that causes a decrease in the potential difference between the two plates by a factor of K.
9. Cequivalent/Ceffective: The total capacitance of a circuit when the capacitors are either in parallel or in series.

Important Formulae:

Capacitance with charge and potential difference	$C=Q/\Delta v$ Units: Farads= (Coulombs)/(Volts)
Capacitance for Parallel Plates when given area of capacitors or distance between two capacitors	$C=Ae_0/d$
Capacitors in Parallel	Qtotal=Q1+Q2, $Q=C\Delta V$, so $C_{eq}\Delta V_{bat}=C1\Delta V1+C2\Delta V2$, Vbat=V1=V2, so: Ceffective= C1+C2
Capacitors in Series	$\Delta V_{bat}=\Delta V1+\Delta V2$, $\Delta V=Q/C$, so (Qbatt)/(Ceq)= (Q1/C1)+(Q2/C2), Qbatt=Q1=Q2, so 1/(Cequivalent)= 1/C2 + 1/C1
Energy in a Capacitor	$\Delta V_{inst}=(q_{inst}/C)$, $dU=\Delta V dq$, $dU=(q_{inst}/C)dq$, $\int dU= {}_0\int^Q(q_{inst}/C)dq$, $U=(Q^2)/(2C)$
Energy in a Capacitor	$U=(CV^2)/2$
Energy in a Capacitor	$U=(QV)/2$

Dielectric	$C=kC_0$
Dielectric in a parallel plate capacitor	$C=kAe_0/d$

Problems:

1.

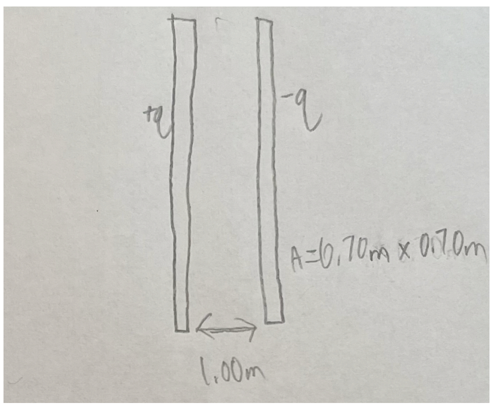
A parallel plate capacitor with two square plates, each of length 0.70m, has a gap of 1.00mm.

a. Calculate the capacitance

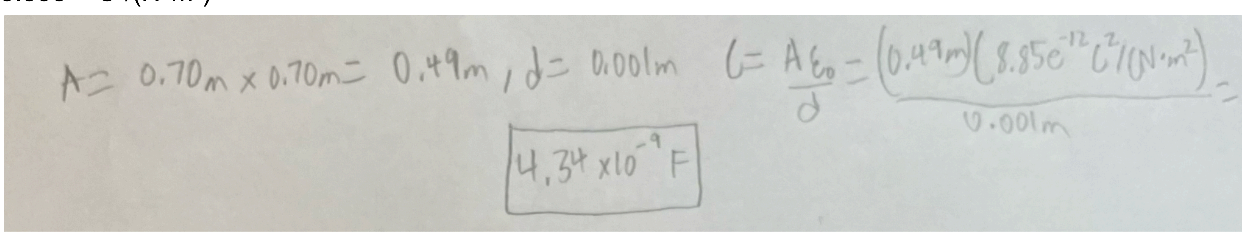
b. A dielectric with a dielectric constant 4.1 is placed in between the plates. Calculate the new capacitance

Answer:

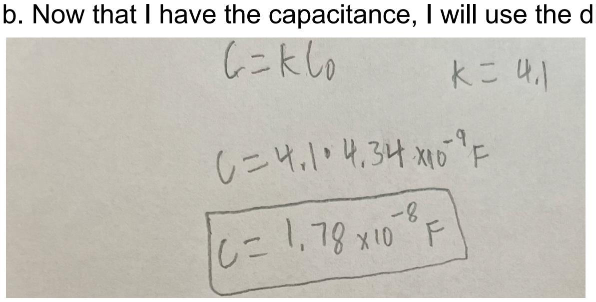
Before everything, draw a diagram of the situation:



a. Because I am given the distance and the area for parallel plates, I will use: $C=Ae_0/d$. e_0 is always $8.85e^{-12}$ $C^2/(N*m^2)$



b. Now that I have the capacitance, I will use the dielectric equation: $C=kC_0$

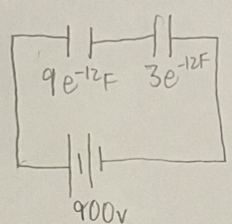


2. A 9.00-pF capacitor is connected in series with an 3.00-pF capacitor and a 900-V potential difference is applied across the pair.

a. What is the charge on each capacitor?

b. What is the voltage across each capacitor?

First draw a diagram!



a. First I will find the $C_{effective}$.

For a capacitor in series:

$$\frac{1}{C_{effective}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eff} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(9e^{-12})(3e^{-12})}{9e^{-12} + 3e^{-12}}$$

$C_{eff} = 2.25 \times 10^{-12} F$ – now that I have the $C_{effective}$, I can plug into $Q = CV$ for charge on every plate

$Q = CV$

$Q = 2.25 \times 10^{-12} \cdot 900$

$\boxed{Q = 2.03 \times 10^{-9} C}$ – across both plates as Q is the same in series.

b. I already have the charge on both plates, and to find their individual voltages, I will plug in their individual capacitances to $V = \frac{Q}{C}$.

$$V = \frac{Q}{C} = \frac{2.03 \times 10^{-9}}{9e^{-12}} = \boxed{225 V}$$

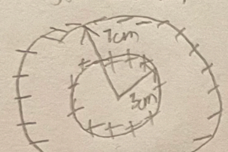
$$V = \frac{Q}{C} = \frac{2.03 \times 10^{-9}}{3e^{-12}} = \boxed{675 V}$$

3. A capacitor has two spheres, one with a radius of 3.00 cm and another with a radius of 7.00 cm.

a. What is the capacitance of the concentric spheres?

b. If the circles are filled with a a material with dielectric constant 4.0, what is the new capacitance?

First draw:



a. As always $C = \frac{Q}{\Delta V}$, but what is ΔV?

$$\Delta V = -\int_{r_i}^{r_f} E\,dr$$

but what is E?

$E = \frac{kq}{r^2}$ derived from Gauss's law

$$\Delta V = -\int_{r_i}^{r_f} E\,dr$$

$$\Delta V = -\int_{r_i}^{r_f} \frac{kQ}{r^2}\,dr$$

$V_f - V_i = -kQ \left. -\frac{1}{r} \right|_{0.07}^{0.03}$ – integration step

$\Delta V = kQ \left(\frac{1}{0.03} - \frac{1}{0.07}\right)$ – plug in bounds

$\Delta V = kQ(19.0) \rightarrow$ Plug back into $C = \frac{Q}{V} = \frac{Q}{kQ(19.0)} = \frac{1}{(8.99e^{9})(19.0)} = \boxed{5.85 pF}$

b. $K = 4.0$

For a dielectric: $C = kC_0$

$C = (4.0)(5.85 \times 10^{-12})$

$\boxed{C = 2.34 \times 10^{-11}}$