Lab: AP Review Sheets **AP Physics**

Chapter 8: Capacitance By: Kate Lim

Background / Summary

This unit focuses on capacitance, which measures a conductor's ability to store charge. There are many applications for capacitance, including circuits and dielectrics.

Definition of Capacitance

"The *capacitance* of a conductor (or conductors) is a measure of the conductor's ability to store charge." (Crash White website)

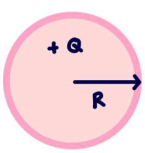
$$C = \frac{Q}{\Delta V} \qquad \text{(units: } [Farads\} = \frac{[Coulombs]}{[Volts]}\text{)}$$

Major Topics

<u>Theoretical capacitances</u>

Isolated conductor: $C = 4\pi\epsilon_0 R$

The capacitance of an isolated conductor is proportionate to the radius, so if the radius is larger, the conductor will be able to hold more charge on its surface. As seen below, the potential does not affect the capacitance as it is ultimately factored out of the final equation.



$C = \frac{Q}{\Delta V}$

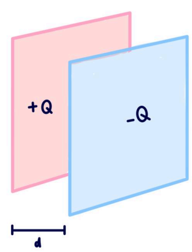
★ the V for an isolated charge is $k\frac{Q}{R}$ & $k = \frac{1}{4\pi\varepsilon_0}$

so... $C = \frac{+Q}{k\frac{Q}{R}} = \frac{\cancel{Q}}{\frac{k\cancel{Q}}{R}} = \frac{R}{k} \rightarrow \boxed{4\pi\varepsilon_0 R}$

Parallel plates: $C = \frac{A\epsilon_0}{d}$

To find the capacitance for two parallel plates—each plate has an area of A, is separated by a distance d, and has equal and opposite charges—you have to find the potential difference and electric field between the plates. *Reminders:

$\Delta V = \int E \cdot ds$ & $E = \frac{\delta}{\epsilon_0}$



$C = \frac{Q}{\Delta V}$

★ for constant E field, $\Delta V = Ed$ & $E = \frac{\sigma}{\varepsilon_0}$ & $\sigma = \frac{q}{A}$

$\Delta V = Ed \rightarrow \Delta V = \frac{\sigma}{\varepsilon_0} d \rightarrow \Delta V = \frac{Qd}{A\varepsilon_0}$

↳ $C = \frac{Q}{\frac{Qd}{A\varepsilon_0}} \rightarrow \boxed{C = \frac{A\varepsilon_0}{d}}$

Cylindrical capacitor:

$C = \frac{L}{2k\ln(\frac{b}{a})} \rightarrow \frac{C}{L} = \frac{1}{2k\ln(\frac{b}{a})}$

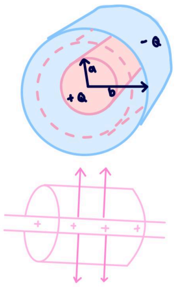
When finding the capacitance of cylindrical capacitors, integrate from high to low potential. The capacitance is dependent on the length of the capacitor, so longer capacitors will have a higher capacitance as they can store more charge.

*Reminders: $\Delta V = -\int_a^b E \cdot dr$ & $E = \frac{2k\lambda}{r}\hat{r}$ & $\lambda = \frac{Q}{L}$

Spherical capacitor: $C = \frac{ab}{k(b-a)}$

For spherical capacitors, use the direction of the electric field to integrate from high to low potential to ensure E and dr are in the same direction. The capacitance for spherical capacitors is similar to that of an isolated conductor, as it is dependent on the inner and outer radii.

*Reminders: $\Delta V = -\int_a^b E \cdot dr$ & $E = k\frac{q}{r^2}\hat{r}$



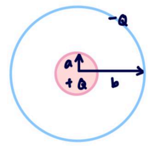
$C = \frac{Q}{\Delta V}$

$\star\ \Delta V = -\int_a^b E \cdot dr$ & $E = \frac{2k\lambda}{r}\hat{r}$ (Guass's law)

$\Delta V = -\int_a^b \frac{2k\lambda}{r}\hat{r} \cdot dr$

$0 - V_a = -2k\lambda \ln r \Big|_a^b \rightarrow V_a = 2k\lambda \ln\left(\frac{b}{a}\right)$

$\hookrightarrow \boxed{C = \frac{L}{2k\ln\left(\frac{b}{a}\right)}}$ $\star\ \lambda = \frac{Q}{L}$



$C = \frac{Q}{\Delta V}$ & $\Delta V = -\int_a^b E \cdot dr$

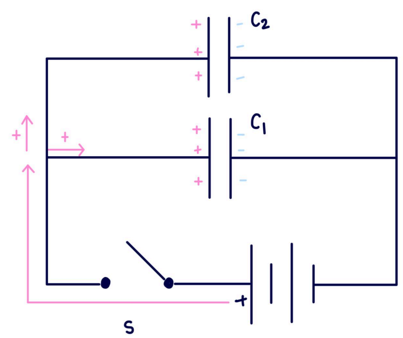
$\star\ E = k\frac{q}{r^2}\hat{r}$ (Guass)

$\Delta V = -\int_a^b k\frac{q}{r^2}\hat{r}\, dr$

$= -kQ\left(\frac{-1}{b} - \frac{-1}{a}\right) = kQ\left(\frac{1}{b} - \frac{1}{a}\right) = kQ\left(\frac{b-a}{ab}\right)$

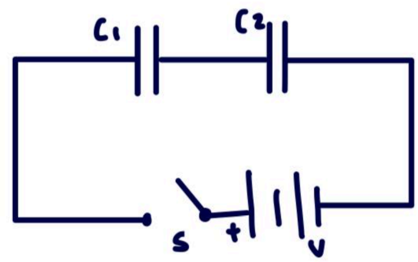
$\hookrightarrow C = \frac{Q}{kQ\left(\frac{b-a}{ab}\right)} \rightarrow \boxed{C = \frac{ab}{k(b-a)}}$

Capacitors in Circuits



Capacitors in *parallel:*

$$C_{effective} = C_1 + C_2$$



Capacitors in *series:*

$$\frac{1}{C_{equivalent}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Electric Potential Energy in a Capacitor

Imagine a circuit where the switch is closed, allowing charge to flow through. To find the work needed to transfer a charge from one plate to the other, do an integration using the capacitance formula as a start.

$$C = \frac{Q}{\Delta V} \rightarrow \Delta V = \frac{Q}{C}$$

$$dU = \Delta V dq \rightarrow dU = \frac{q_{inst}}{C} dq$$

$$\int dU = \int_0^Q \frac{q_{inst}}{C} dq \Rightarrow U = \frac{1}{2}\frac{Q^2}{C}$$

and… $U = \frac{1}{2}\frac{Q^2}{C}$ & $Q = C\Delta V$

so $U = \frac{1}{2}CV^2 \Rightarrow U = \frac{1}{2}QV$

Dielectrics

Dialectrics, insulators that are inserted between a capacitor's plates, can affect the capacitance of a system. For isolated capacitors:

1. The charge on the plates doesn't change
2. The bound charges in the insulators are polarized → creates an electric field that produces a net electric field smaller than before
3. The electric potential is reduced by the dielectric constant K

$$V = \frac{V_0}{K}$$

4. The capacitance of the system increases because V decreases by $\frac{V_0}{K}$, so $C = \frac{Q_0}{\frac{V_0}{K}} \rightarrow k\frac{Q_0}{V_0} \rightarrow$

$$C = kC_0$$

*For parallel-place capacitors: $C_{\parallel} = k\frac{A}{d}\epsilon_0$

Important Formulae

Solving for capacitance: $C = \frac{Q}{\Delta V}$	Capacitance for parallel circuits: $C_{effective} = C_1 + C_2$
Capacitance for an isolated conductor: $C = 4\pi\epsilon_0 R$	Capacitance for circuits in series: $\frac{1}{C_{equivalent}} = \frac{1}{C_1} + \frac{1}{C_2}$
Capacitance for parallel plates: $C = \frac{A\epsilon_0}{d}$	Potential energy of a capacitor: $U = \frac{1}{2}CV^2 \Rightarrow U = \frac{1}{2}QV$
Capacitance for a cylindrical capacitor: $C = \frac{L}{2kln(\frac{b}{a})} \rightarrow \frac{C}{L} = \frac{1}{2kln(\frac{b}{a})}$	Capacitance with a dielectric: $C = kC_0$
Capacitance for a spherical capacitor: $C = \frac{ab}{k(b-a)}$	Parallel-place capacitors with a dielectric: $C_{\parallel} = k\frac{A}{d}\epsilon_0$

Problems:

1. What capacitance is needed to store 3.00μC of charge at a voltage of 120 V? (#23 from the textbook)

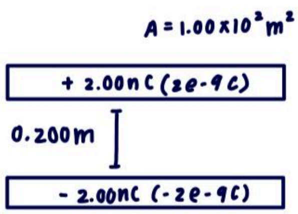
$Q = 3.00\mu C \cdot \frac{1\,C}{10^6 \mu C} = 3.00e\text{-}6\ C$

$V = 120\ V$

$\rightarrow C = \frac{Q}{\Delta V} = \frac{3.00e\text{-}6}{120} = \boxed{2.5e\text{-}8\ F}$

In this problem, the charge (Q) and potential (V) were given in the problem. To find the capacitance, use the equation $C = \frac{Q}{\Delta V}$ and plug in values.

2. An anxious physicist worries that the two metal shelves of a wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them, given the empty shelves have an area of $1.00\text{x}10^2$ m^2 and are 0.200 m apart? To show that this voltage poses a small hazard, calculate the energy stored. (based on #47 from textbook)



first find capacitance for parallel plates: $C = \frac{A\varepsilon_0}{d}$

$C = \frac{A\varepsilon_0}{d} = \frac{(1.00\text{x}10^2)(8.85\text{x}10^{-12})}{0.2} = 4.425\text{x}10^{-9}\text{F}$

↳ then solve for voltage using $C = \frac{Q}{\Delta V} \rightarrow \Delta V = \frac{Q}{C}$

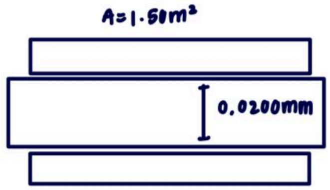
$\Delta V = \frac{(2e-9)}{4.425e-9} = \boxed{0.452\text{ V}}$

find energy stored: $U = \frac{1}{2}CV^2$

$C = 4.425e-9$, $V = 0.452$ $\rightarrow U = \frac{1}{2}(4.425e-9)(0.452) = \boxed{4.52\text{ e-}10\text{ J}}$

To find the voltage, you first need to find the capacitance using the equation for parallel plates $C = \frac{A\epsilon_0}{d}$. After finding the capacitance, you can solve for the voltage using the basic capacitance equation $C = \frac{Q}{\Delta V}$. After you find the volate, you can plug the values above into the equation $u = \frac{1}{2}CV^2$ to find the energy stored.

3. (a) What is the capacitance of a parallel-plate capacitor with plates of area 1.50 m^2 that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?



$A = 1.50\,m^2$

$d = 0.0200\,mm \cdot \frac{1\,m}{1000\,mm} = 0.02e\text{-}3\,m$

$V = 9.0V$

a) $C = ?$

capacitance for parallel plate capacitors: $C_{\parallel} = \frac{A}{d}\epsilon_0$

↳ $C_{\parallel} = \frac{(1.5)}{0.02e\text{-}3}(8.85e\text{-}12) = \boxed{6.64e\text{-}7\ F}$

b) $Q = ?$

$C = \frac{Q}{\Delta V} \rightarrow Q = C\Delta V$

$= (6.64e\text{-}7)(9) = \boxed{5.98e\text{-}6\ C}$

First find the capacitance using the parallel-place capacitance equation: $C_{\parallel} = k\frac{A}{d}\epsilon_0$

After finding the capacitance, you can solve for the charge by using the basic capacitance equation and rearranging to solve for Q.