Lab: AP Review Sheets

Chapter 10: DC Circuits

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1 Background

This chapter concerns the movement of charge through simple DC (direct current) circuits containing resistors and capacitors.

2 Vocabulary

- Circuit: In a circuit, components (batteries, resistors, capacitors, inductors, lightbulbs, etc) are connected by conductive wires or traces through which current can flow.
- EMF (ϵ or E, Volts (V)): An EMF (also written as emf), or *electromotive force*, is a potential difference where current is flowing.
- Terminal Voltage ($V_{terminal}$, Volts (V)): The potential across the terminals of a battery when the battery is powering a circuit. The terminal voltage of a battery is lower than the battery's nominal voltage due to the battery's internal resistance.
- Resistor (R, Ohms (Ω)): Components which restrict the flow of current (I).
- Capacitor (C, Farads (F)): Components which store energy by accumulating charges two closely spaced surfaces insulated from each other.

3 Topics

EMF & Terminal Voltage

When a circuit is "open" (current is flowing through the circuit), the battery is providing an *emf*, an electromotive force. This electromotive force E (also written as ϵ) is measured in *volts*.

When a battery is in a circuit through which current is flowing, the battery's *terminal voltage* (voltage across the battery terminals) is less than the battery's *nominal voltage* (the voltage of the battery when it is not connected to a circuit). This is due to the *internal resistance* (r) of the battery, which is not to be confused with the resistance of the actual circuit, denoted using R.

$$V_{terminal} = E - Ir$$

Resistors in Series & Parallel

Resistors restrict the flow of current through a circuit. The following equation models the effect of a resistor on a simple circuit:

$$I = \frac{\Delta V}{R}$$

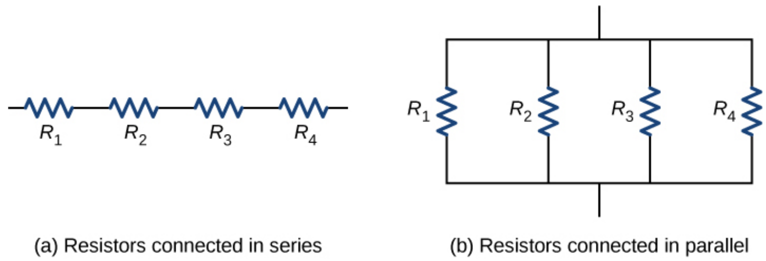

Figure 1: Resistors in Series and Parallel

Resistors can be arranged in *series* and in *parallel*.

Resistors in series all maintain the same original current I but the voltage across each resistor in series varies according to $\Delta V = IR$. Resistors in series produce a total resistance of:

$$R_{eq} = R_1 + R_2 + ...$$

Resistors in parallel share the same potential difference but the current varies according to $I = \Delta V R$. Resistors in parallel produce a total resistance of:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

Kirchhoff's Rules

The two important rules are the Junction Rule and Loop Rule.

Junction Rule: "The total current traveling into a node (a connecting point for 3 or more wires) = the total current coming out of the node"

$$\sum I_{in} = \sum I_{out}$$

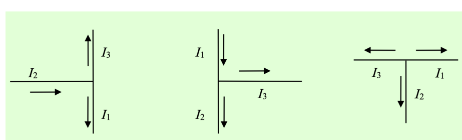

Figure 2: Junction Rule

Loop Rule: "The sum of the potential changes around a closed loop must equal zero"

$$\sum \Delta V = 0$$

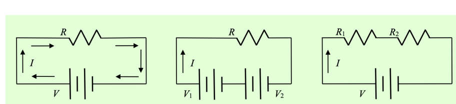

Figure 3: Loop Rule

RC Circuits

RC circuits contain a resistor and capacitor in series. When current is flowing, charges build up on the capacitor until the capacitor is full, at which point current is 0.

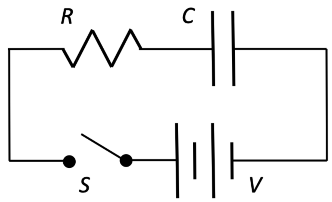

Figure 4: RC Circuit

The accumulation of charges on a charging capacitor is modeled using a time constant τ, which we obtain using $\tau = RC$. We use this time constant in the following equation to determine the charge of a charging capacitor at a given time:

$$q = CV_0(1 - e^{-\frac{t}{\tau}})$$

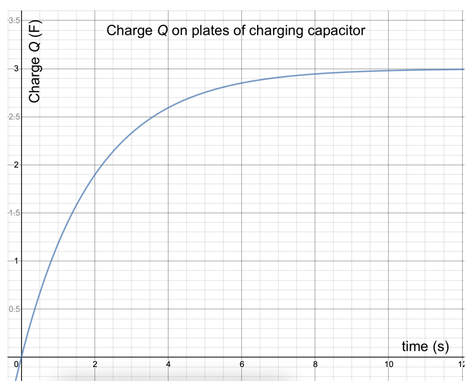

Figure 5: Charging Capacitor

4 Formulae

Terminal Voltage: $V_{terminal} = \epsilon - Ir$

Resistors in Series: $R_{eq} = R_1 + R_2 + ...$

Resistors in Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$

Junction Rule: $\sum I_{in} = \sum I_{out}$

Loop Rule: $\sum V = 0$

Charge of Charging Capacitor: $q = CV_0(1 - e^{-\frac{t}{\tau}})$

Current during Charging Capacitor: $I = \frac{\epsilon}{R} e^{-\frac{t}{RC}}$

Voltage of Charging Capacitor: $V(t) = V(1 - e^{-\frac{t}{\tau}})$

Time Constant: $\tau = RC$

5 Problems

5.1 Problem 1

The terminal voltage of a 15 V battery is 9 volts when providing 20 watts of power to an external load resistor R. Determine R and the internal resistance of the battery.

Solution:

We can first calculate I using $P = V_t \times I$, which gives $I = \frac{20}{9}$ amps. Then, using Ohm's law, $V_t = I \times R =$, we can determine $R = \frac{V_t}{I} = \frac{81}{20} = 4.1$ ohms.

Now we can determine r using $V_{terminal} = \epsilon - Ir$ by rearranging to $r = \frac{\epsilon - V_t}{I} = \frac{27}{20} = 1.4$ ohms.

5.2 Problem 2

Express the total resistance of this configuration of resistors in terms of $R1$.

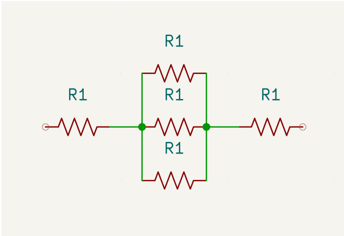

Figure 6: Problem 2 Circuit

Solution:

We can treat the three resistors in parallel like one single resistor using the equation:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

Using this equation, the resistance of the three parallel resistors is $(\frac{1}{R1} + \frac{1}{R1} + \frac{1}{R1})^{-1}\Omega$.

Now, we can add the resistances of the two resistors in series, giving us a total resistance of $2 * R1 + (\frac{1}{R1} + \frac{1}{R1} + \frac{1}{R1})^{-1}\Omega$.

5.3 Problem 3

An RC circuit contains a resistor $R = 10\,\mathrm{k\Omega}$, an uncharged capacitor $C = 2\,\mathrm{\mu F}$, and a $12\,\mathrm{V}$ battery. Determine the time constant and voltage across the capacitor after 10 milliseconds.

Solution:

The time constant is given by $\tau = RC$, giving $\tau = 20 \times 10^{-3}\,\mathrm{s}$.

The voltage across a charging capacitor is given by $V(t) = V(1 - e^{-\frac{t}{\tau}})$. Since $t = 10\,\mathrm{ms}$, we can directly substitute into this equation.

$$V(t) = V(1 - e^{-\frac{t}{\tau}})$$
$$V(5\tau) = 12(1 - e^{-\frac{10\,\mathrm{ms}}{20\,\mathrm{ms}}})$$
$$V(5\tau) = 12(1 - e^{-0.5})$$
$$V(5\tau) = 4.72\,\mathrm{V}$$