Lab: AP Review Sheets
Chapter 6: Gauss's Law

AP Physics

by Roy Otamura

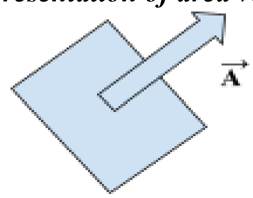
Background/Summary

This unit introduces the concept of flux, which relates to a physical quantity and an area through which it is flowing. To calculate electric flux, this unit derives Gauss's law and describes Gaussian shapes.

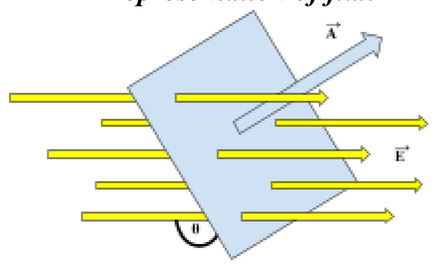
Key Terms and Vocabulary

1. $\vec{E}$ represents the vector of the electric field, both direction and magnitude.
2. $\vec{A}$ represents the area of a surface and the normal vector perpendicular to the surface.
3. For an area vector where $\theta < 90^\circ$, flux is positive. When $\theta < 90^\circ$, flux is negative.
4. k is a constant equal to $\frac{1}{4\pi\varepsilon_0}$ or $8.99 \times 10^9\ \mathrm{N}\frac{m^2}{C^2}$
5. A Gaussian surface has flux through at least some of its surface that is perpendicular to that area. The Gaussian surface may have areas with no flux. This occurs when the area and the electric field are perpendicular because cos(90) = 0.
6. There is no Gaussian surface to properly analyze situations such as an electric dipole
7. Without any enclosed charge, net flux is always zero regardless of electric fields

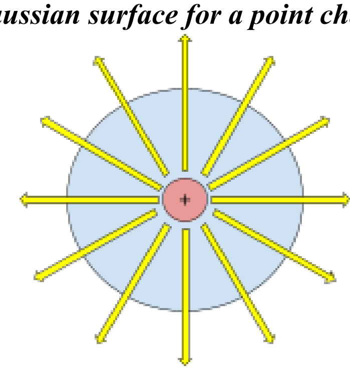
Representation of area vector



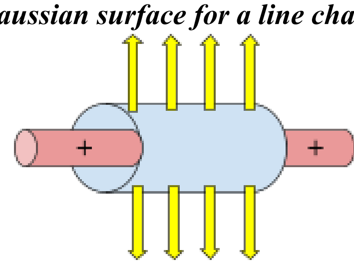
Representation of flux



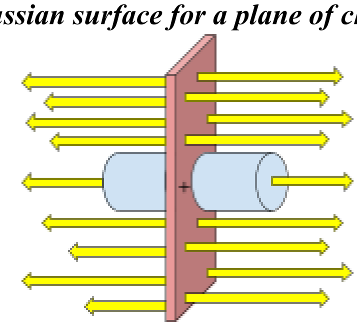
Gaussian surface for a point charge



Gaussian surface for a line charge



Gaussian surface for a plane of charge



Relevant Formulae

$\Phi_e = \oint \vec{E} \bullet \vec{dA}$

$\Phi_e = EAcos(\theta)$

$\vec{E}_P = \frac{kq}{r^2}\hat{r}$

$F_E = k\frac{q_1 q_2}{r^2}$

$\Phi_{\text{point charge}} = \oint \vec{E} \bullet \vec{dA}$

$= \oint E \bullet dA \cos(90)$

$= E \oint dA$

$= (\frac{kq}{r^2})(4\pi r^2)$

$= 4\pi kq$

$\Phi_{\text{point charge}} = \frac{q_{in}}{\varepsilon_0}$

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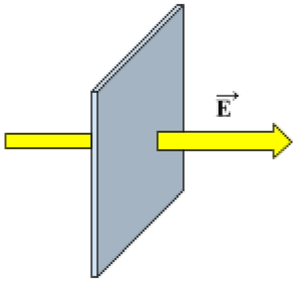
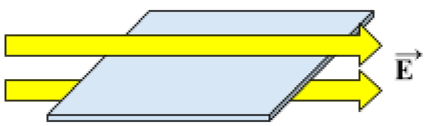
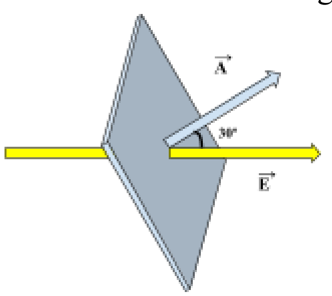
1. Find the electric flux through a 25 cm by 30 cm plate from an electric field of 40 N/C when:
    a. The field is perpendicular to the plate.



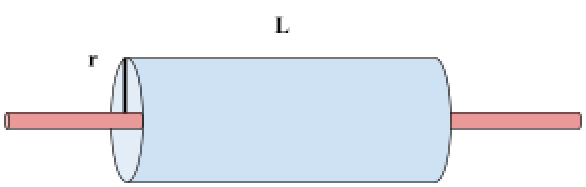
    b. The field is parallel to the plate (consider the width of the plate to be zero).



    c. The field makes a 30º angle with the normal to the area.



2. An infinite charged wire with charge per unit length λ lies on the central axis of a cylindrical surface of radius r and length L. What is the electric field at the surface of the cylinder?

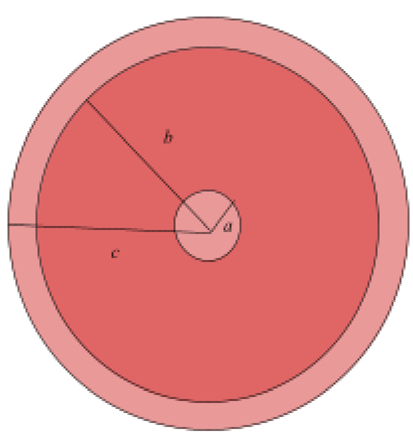


3. A non-conducting sphere with uniform charge density with charge $+q$ with radius a is inside a non-conducting shell of inner radius b and outer radius c that has a uniformly-distributed charge $+2q$. Calculate the electric field a distance r away from the center when:

    a. $r < a$

    b. $r < a < b$

    c. $b < r < c$



Solutions

1. The plate is 25 cm by 30 cm, or 0.25 m by 0.30 m. The area of the plate is $0.25 \times 0.30 = 0.075$ m^2.

   a. To find flux we use $\Phi_e = \oint \vec{E} \bullet \vec{dA} = EA\cos(\theta)$. The field is perpendicular to the plate, so the θ between the field and the area normal vector is 0º.

   $\Phi_e = EA\cos(\theta)$

   $\Phi_e = (40\ \text{N/C})(0.075\ \text{m}^2)\cos(90)$

   $\Phi_e = 3.0\ \text{Nm}^2/\text{C}$

   b. We again use the same concept that $\Phi_e = EA\cos(\theta)$. This time, the angle is 90º so cos(θ) = 0. Thus, the flux through the area is 0 Nm^2/C.

   c. The angle between the area normal and the field is 30º, so when using $\Phi_e = EA\cos(\theta)$, θ = 30º.

   $\Phi_e = (40\ \text{N/C})(0.075\ \text{m}^2)\cos(30)$

   $\Phi_e = 2.6\ \text{Nm}^2/\text{C}$

2. Once again we use $\Phi_e = \oint E \bullet dA$, but we also use $\Phi_e = \frac{q_{in}}{\varepsilon_0}$. The electric field goes out radially from the wire, so it is always perpendicular to the surface of the cylinder where it is passing through. As such, cos(θ) is simply one and the vectors do not affect the calculation of flux. The ends of the cylinder are parallel to the electric field and do not contribute to the flux. So the $\oint dA$ is the length L of the cylinder times circumference of the end, or $2\pi r$.

   $\Phi_e = \oint E \bullet dA = EA = \frac{q_{in}}{\varepsilon_0}$

   $E(\text{L})(2\pi r) = \frac{q_{in}}{\varepsilon_0}$

   $q = \lambda L$

   $E = \frac{\lambda L}{2L\pi r} = \frac{2\lambda k}{r}$

Solutions

3. Use Gauss's Law, $\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$, to find electric fields at various radii.

   a. $r < a$

   $\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$

   $EA = \frac{q_{in}}{\varepsilon_0}$

   Because q_{in} depends on the radius, we have to use the charge density to calculate the q_{in}.

   $\rho = \frac{Q}{V} = \frac{q_{in}}{V_{in}}$

   $q_{in} = \frac{QV_{in}}{V} = \frac{Q(\frac{4}{3}\pi r^3)}{\frac{4}{3}\pi a^3} = \frac{Qr^3}{a^3}$

   $E(4\pi r^2) = \frac{Qr^3}{a^3\varepsilon_0}$

   $E = \frac{Qr}{4\pi a^3\varepsilon_0} = \frac{kqr}{a^3}$

   b. $r \le a \le b$

   $\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\varepsilon_0} = \frac{q_{sphere} + q_{shell}}{\varepsilon_0}$

   q_{shell} depends on the radius, so again we use charge density to calculate q_{shell}.

   $\rho = \frac{Q}{V} = \frac{q_{shell}}{V_{she;;}}$

   $q_{shell} = \frac{QV_{shell}}{V_{tot}} = \frac{Q(\frac{4}{3}\pi(r^3 - b^3))}{\frac{4}{3}\pi(c^3 - b^3)} = \frac{2q(r^3 - b^3)}{c^3 - b^3}$

   $E(4\pi r^2) = \frac{q + \frac{2q(r^3 - b^3)}{c^3 - b^3}}{\varepsilon_0}$

   $E = \frac{kq}{r^2} + \frac{2kq(r^3 - b^3)}{r^2(c^3 - b^3)}$

   c. $b < r < c$

   Outside the sphere, it basically becomes a point charge.

   $\Phi_e = \oint E \bullet dA = \frac{q_{in}}{\varepsilon_0}$

   $EA = \frac{q_{in}}{\varepsilon_0}$

   $E(4\pi r^2) = \frac{3q}{\varepsilon_0}$

   $E = \frac{3q}{4\pi\varepsilon_0 r^2} = \frac{3kq}{r^2}$