Brendan Poon

Chapter 10: Direct Current Circuits

Background and Major Topics

This unit explores the characteristics of more complex circuits than previous units. Problems require a strong understanding of the functions of various circuit components. Major topics include internal resistance, resistors in series and parallel, Kirchhoff's Junction and Loop Rules, and RC Circuits.

Vocabulary

The **electromotive force (emf, ε)** is the potential difference of a battery disconnected from a circuit. The **terminal voltage** is the potential difference of a battery connected to a circuit. The terminal voltage is less than the electromotive force because of **internal resistance**: an impedance to current within the battery itself that causes a small voltage drop across the battery.

Resistors in series increase total resistance and experience the same current but different voltages. **Resistors in parallel** decrease total resistance by creating additional pathways for current to flow and experience different currents but the same voltage.

Kirchhoff's Rules enable analysis of complex circuits by arbitrarily assigning current directions. The **Junction Rule** states that the current flowing into a junction equals the current flowing out of it due to Conservation of Charge. (A junction is a node where three or more wires connect.) The **Loop Rule** states that the sum of the potential lifts and drops around a closed loop equals zero because of Conservation of Energy. Note that when an assigned current flows backwards relative to conventional current, components that typically cause voltage drops (e.g., resistors) cause voltage lifts, and vice versa.

An **RC Circuit** is a circuit with a resistor and a capacitor in series. When a battery is connected to an RC Circuit, the charge on the capacitor increases over time, while the current through the circuit decreases over time because the potential difference across the capacitor "opposes" the battery. When no battery is connected and the charged capacitor provides the potential difference, both the charge on the capacitor *and* the current decrease over time because the resistor dissipates the charge on the capacitor. Equations describing charge and current in an RC Circuit can be derived using Kirchhoff's Loop Rule and separation of variables.

Formulae

$$V_t = \mathcal{E} - Ir$$

$$R_s = \sum_i R_i \qquad \frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

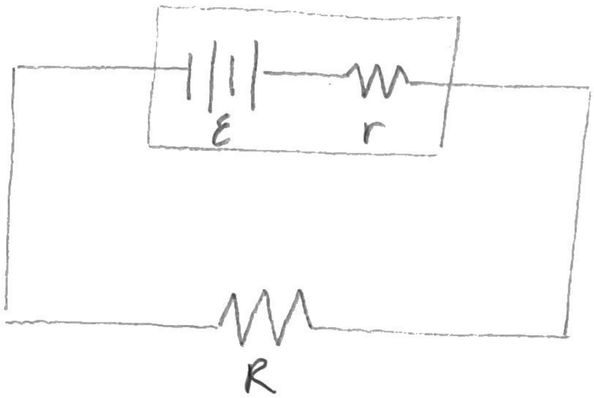
$$\sum I_{in} = \sum I_{out} \qquad \sum \Delta V = 0$$ (Apply after assigning currents)

$$\tau = RC \qquad Q = CV_0\left(1 - e^{-t/\tau}\right) \qquad I = \frac{V_0}{R} e^{-t/\tau}$$ (Charging)

$$Q = CV_0 e^{-t/\tau} \qquad I = -\frac{Q_0}{\tau} e^{-t/\tau}$$ (Discharging)

Diagrams

1. Internal resistance

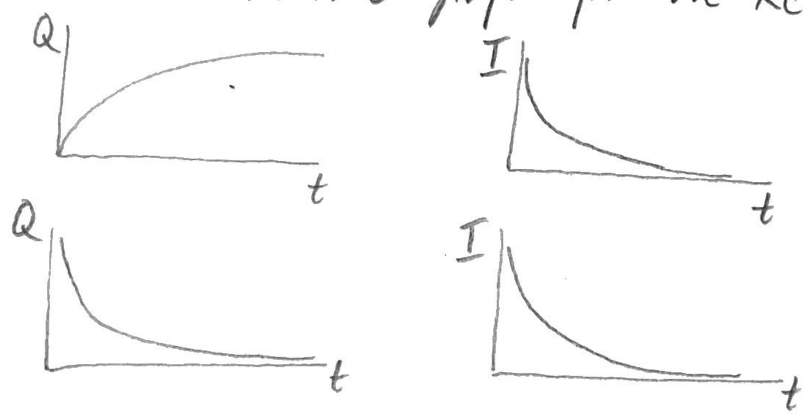


The box indicates the battery.
Note that $\mathcal{E} - Ir = IR$.

2. Charge and current vs. time graphs for an RC Circuit

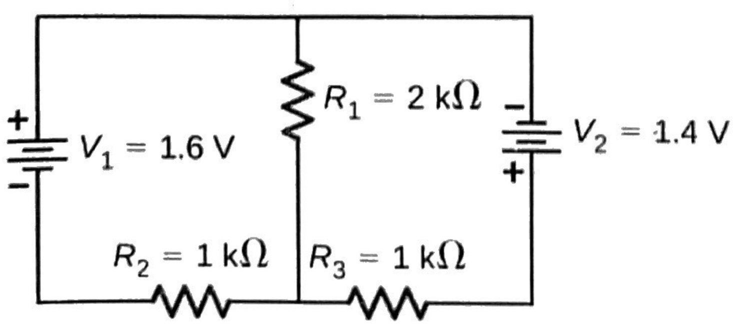
Charging

Discharging



Problems

1. (Easy) What is the internal resistance of a voltage source if its terminal potential drops by 2.00 V when the current supplied increases by 5.00 A? Can the emf of the voltage source be found with the information supplied?
2. (Medium) A 500-Ω resistor, an uncharged 1.50-μF capacitor, and a 6.16-V emf are connected in series. What is the initial current? What is the time constant? What is the current after one time constant?
3. (Hard) Consider the circuit shown below. What is the current through each resistor? What is the power dissipated by the circuit? What is the power supplied to the circuit?



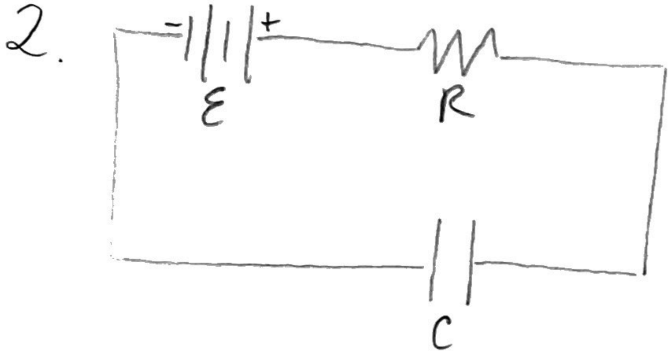
Solutions

1. $V_t = \mathcal{E} - Ir \rightarrow \Delta V_t = Ir \rightarrow 2 = 5r \rightarrow r = \boxed{0.400\ \Omega}$

The emf $\boxed{\text{cannot}}$ be found – we have one equation with two unknowns.

Another way to justify this is that we only have information about changes in potential, not absolute potential.

2.



$\mathcal{E} = 6.16\ V$
$R = 500\ \Omega$
$C = 1.50 \cdot 10^{-6}\ F$

When current first begins to flow, no charge has built up on the capacitor yet, so it can effectively be ignored. Thus, the initial current is given by Ohm's Law:

$$V = IR \rightarrow I = \frac{V}{R} = \frac{6.16}{500} = \boxed{1.23 \cdot 10^{-2}\ A}$$

The time constant only depends on the fixed characteristics of the circuit components:

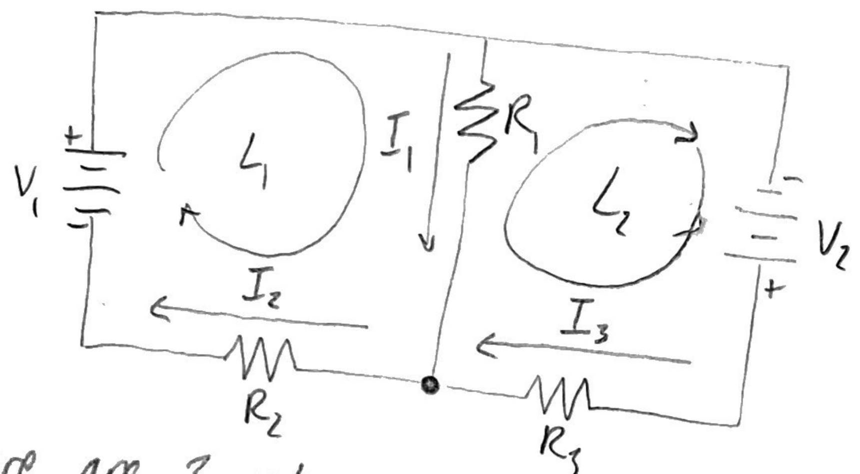
$$\tau = RC = (500)(1.50 \cdot 10^{-6}) = \boxed{7.50 \cdot 10^{-4}\ s}$$

The capacitor is initially uncharged. Over time, as charge accumulates, current decreases according to:

$$I = I_{max} e^{-t/\tau} = (1.23 \cdot 10^{-2}) e^{-t/\tau}$$

After one time constant, $t = \tau$, so $I = (1.23 \cdot 10^{-2}) e^{-1} = \boxed{4.53 \cdot 10^{-3}\ A}$

3.



Arbitrarily assign current directions to each resistor.

Apply the Junction Rule:

$\Sigma I_{in} = \Sigma I_{out} \rightarrow I_1 + I_3 = I_2$

Note that the unmarked junction would yield the same eq'n.

There are 3 unknowns, so we need 2 more eq'ns from the Loop Rule, $\Sigma \Delta V = 0$.

For L_1: $V_1 - I_1 R_1 - I_2 R_2 = 0 \rightarrow 1.6 - 2000 I_1 - 1000 I_2 = 0$

For L_2: $V_2 - I_3 R_3 + I_1 R_1 = 0 \rightarrow 1.4 - 1000 I_3 + 2000 I_1 = 0$

Re-arrange the eq'ns into the same form:

$I_1 - I_2 + I_3 = 0$ $\quad 2000 I_1 + 1000 I_2 + 0 I_3 = 1.6$ $\quad 2000 I_1 + 0 I_2 - 1000 I_3 = -1.4$

Row-reduce the coefficient matrix:

$$\text{rref}\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2000 & 1000 & 0 & 1.6 \\ 2000 & 0 & -1000 & -1.4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4E^{-5} \\ 0 & 1 & 0 & 0.00152 \\ 0 & 0 & 1 & 0.00148 \end{bmatrix} \rightarrow \boxed{\begin{aligned} I_1 &= 4.0 \cdot 10^{-5}\ A \\ I_2 &= 1.52 \cdot 10^{-3}\ A \\ I_3 &= 1.48 \cdot 10^{-3}\ A \end{aligned}}$$

By Conservation of Energy, $P_{dissipated} = P_{supplied} = I_3 V_2 + I_2 V_1 = (1.48 \cdot 10^{-3})(1.4) + (1.52 \cdot 10^{-3})(1.6) = \boxed{4.50 \cdot 10^{-3}\ W}$