Lab: AP Review Sheets **AP Physics**
Chapter 6: Gauss's Law **By Carolyn Wu**

Background / Summary
In this unit, we learned how Gauss's Law is a strategy to determine the electric field around charges. This strategy is effective in certain situations, and cannot be applied to all situations. An understanding of flux is also needed to apply Gauss's Law.

Major Topics:
Electric Flux
Definition: The amount of field, taking into account both magnitude and direction, that's passing through a given area.
- As the magnitude of the electric field increases, so does the flux.
- Flux is a dot product, so you must take into account the angle

Equation: $\Phi_e = \vec{E} \cdot \vec{A}$
$\Phi_e = \vec{E} A \cos\theta$

$$\Phi_e = \int \vec{E} \cdot d\vec{A}$$

If field lines are in varying orientations, then the original equation changes to take into account that variation: ^

Note: The permittivity of free space: $k = \frac{1}{4\pi \varepsilon_0} \rightarrow \varepsilon_0 = \frac{1}{4\pi k}$

Gauss's Law: Definition (from crashwhite): "The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0."

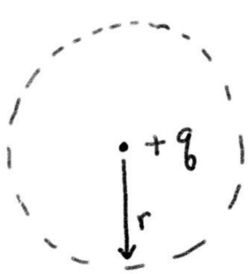
To solve Gauss's problems, we would first imagine that the "closed surface" contains the internal charge and resides at the point where we want to find the net flux. "Closed surface" = Gaussian surface.

Equation that finds flux through a Gaussian surface: $\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

Then using this equation, we can also find the electric field at the point at which the Gaussian surface resides.
How to determine the Gaussian surface:
1. The surface needs to be symmetric to the object, whether it's a sphere, hoop, line, cylinder etc
2. Field lines need to pass perpendicularly through the Gaussian surface
3. Gaussian surface needs to intersect the point at which we are trying to find the electric field
4. There can be an area on the shape that is parallel to the field lines, but it's inconsequential so you don't include it in the calculation

How to use Gauss's Law for key scenarios:
** Always start with the base equation, then plug the specific area for your situation into that equation

The electric field of a point charge:



start w/ Gauss's law

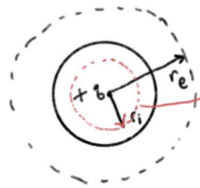
$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\vec{E} \oint d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\vec{E}(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$$

$$\vec{E}(4\pi r^2) = \frac{q_{in}}{\frac{1}{4\pi k}} = 4\pi k q_{in} = \boxed{\frac{kq}{r^2}}$$

<u>Spherically symmetric charge distribution:</u>
If the Gaussian surface is outside the sphere with a uniform charge density, then the equation for an electric field remains the same since all charge is internal. But if the Gaussian surface is inside the sphere:



for gaussian surface inside the charge distribution
↳ we only want charge <u>internal</u>

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$\vec{E} \oint d\vec{A} = 4\pi k q_{in}$

$E(4\pi r^2) = 4\pi k q_{in}$

$E = k\frac{q_{in}}{r^2}$

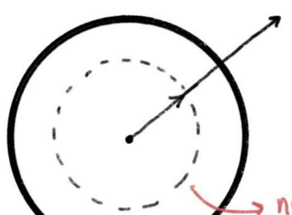
→ now calc q_{in} as a fraction of Q

$\rho = \frac{Q}{V_{total}} = \frac{q_{in}}{V_{in}}$

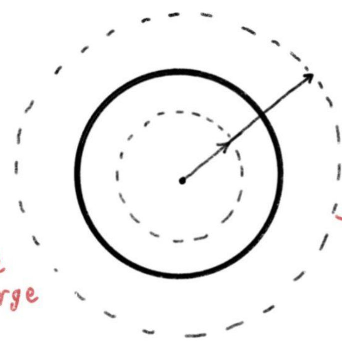
$q_{in} = \frac{V_{in}}{V_{total}} Q = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} Q$

sub in → $E = k\frac{Qr}{a^3}$

<u>E field of a thin, spherical shell:</u>

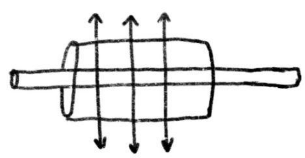


no electric field @ this point because no net internal charge



same electric field as point charge since it's internal charge

<u>E field of a line charge:</u>
It's important to note that the lines extend radially out from the line, but from a side view you only see it go up and down. To create a Gaussian shape that all field lines are perpendicular to, you use the walls of a cylinder. Do not include the caps of the cylinder since the field lines are parallel to the plane of those ends:



start w/ $\Phi_e = \oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}$

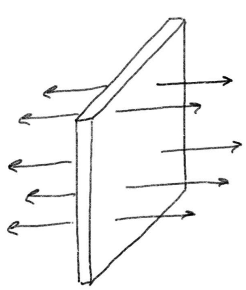
$Q = \lambda L$

$E \oint dA = (\lambda L)(4\pi k)$

$E(2\pi r L) = (\lambda L)(4\pi k)$

$E = 2k\frac{\lambda}{r}$

<u>E field of a nonconducting plane of charge:</u>



use $\sigma = Q/A$

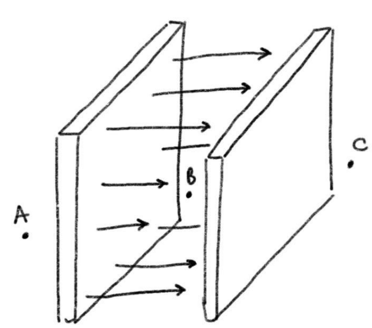
$\oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}$

$E \oint dA = \frac{\sigma A}{\varepsilon_0}$

$E 2A = \frac{\sigma A}{\varepsilon_0}$

$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

<u>E field near parallel planes of charge:</u>
You start with the base electric field we derived right above with a nonconducting plane of charge, but because there are two planes, one positive and one negative, the electric fields of those two planes interact and produce new electric fields:



@ A, B, & C there is an electric field of $E = \frac{\sigma}{2\varepsilon_0}$

but electric field @ A & C cancels out so it's zero

but field at B doesn't cancel out, actually it doubles

→ $E = \frac{\sigma}{\varepsilon_0}$

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Important key phrases / key points:

- Electric flux measures the amount of field passing through a given area
- Electric flux relies on the net charge. No net charge = no net flux
- To use Gauss's Law, the situation has to be symmetric. i.e. spheres, long cylinders. A dipole, since it's not symmetrical, would not work with Gauss's Law
- If field lines are parallel with the plane of the gaussian surface, then that surface should not be included in the analysis

Important formulae/Values:

using Efield to get back to coulomb's

$$E = \frac{kq}{r^2} \rightarrow E = \frac{F}{q} \rightarrow \text{so } F = k\frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_0} \rightarrow \varepsilon_0 = \frac{1}{4\pi k}$$

K = 8.99 e9 Nm^2/C^2
ε_0 = 8.85 e-12 C^2/(Nm^2)

$$\rho = \frac{Q}{V} \quad \sigma = \frac{Q}{A}$$

$$\lambda = \frac{Q}{L}$$

Use these when told "charge per unit length" "Uniformly distributed with __ density" etc

Finding electric flux when given area and E field

$$\Phi_e = \vec{E}\cdot\vec{A}$$

$$\Phi_e = \vec{E}A\cos\theta$$

Gauss's Law to find E field for symmetric situations

$$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

Equation for E field between parallel plates of charge

$$E = \frac{\sigma}{\varepsilon_0}$$

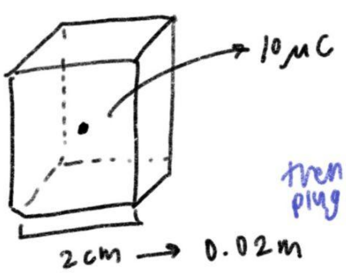
Problems:

1. [Easy] A point charge of 10μC is at an unspecified location inside a cube of side 2 cm. Find the net electric flux through the surfaces of the cube.
2. [Medium] Charge is distributed uniformly with a density ϱ throughout an infinitely long cylindrical volume of radius R. Show that the field of this charge distribution is directed radially with respect to the cylinder and that $E = \frac{\rho r}{2\varepsilon_0} \quad (r \leq R);$ & $E = \frac{\rho R^2}{2\varepsilon_0 r} \quad (r \geq R).$

3. [Hard] A long cylinder of copper of radius 3 cm is charged so that it has a uniform charge per unit length on its surface of 3 C/m. (a) Find the electric field inside and outside the cylinder. (b) Draw electric field lines in a plane perpendicular to the rod.

Solutions + blurbs:

1)



first set up electric flux equation

$$\Phi_e = \frac{q_{in}}{\varepsilon_0}$$

then plug in the charge given

$$= \frac{10\times10^{-6}}{\frac{1}{4\pi k}}$$

and $\varepsilon_0 = \frac{1}{4\pi k}$

rearrange and plug in the value for k

$$(10\times10^{-6})(4\pi(8.99\times10^9))$$

$$= 1129716.7$$

$$= \boxed{1.13\times10^6 \ \frac{Nm^2}{C}}$$

This problem doesn't rely so much on Gauss's Law, as it does just on the concept of electric flux. Electric flux is dependent on the internal charge, and we can solve this problem just by using the given equation for electric flux of charge internal divided by the Vacuum permittivity constant. We plug in our known values, and then just solve to get the final answer.

2)

density ρ

we know that $\rho = \frac{Q}{V}$

$V\rho = Q$

but we want Qint

$Q_{int} = V_{int}\rho$

so we need Vint

$V_{int} = \pi r^2 l$

& use internal volume

a) For $r \le R$

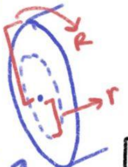
start with $\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$ equation for gauss's law

$E\oint dA = \frac{q_{in}}{\varepsilon_0}$

plug $V_{int}\rho$ into here

$E(2\pi r l) = \frac{\pi r^2 l \rho}{\varepsilon_0}$

surface area of the gaussian surface which in this case is inside



$E = \frac{\pi r^2 l \rho}{2\pi r l \varepsilon_0}$

$\boxed{E = \frac{r\rho}{2\varepsilon_0}}$

still start w/

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$E\oint dA = \frac{q_{in}}{\varepsilon_0}$

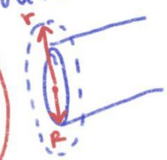
$E(2\pi r l) = \frac{\pi R^2 l \rho}{\varepsilon_0}$

dA remains the same since surface area of a cylinder is the same

$E = \frac{\pi R^2 l \rho}{2\pi r l \varepsilon_0}$

$\boxed{E = \frac{R^2\rho}{2r\varepsilon_0}}$

Same situation but now gaussian surface is on the outside



so now q_{in} is $V_{int}\rho$ & $V_{int} = \pi R^2 l$

First establish that "charge is distributed uniformly with a density ϱ," and we know that ϱ = Q/V (we'll need this info later). Then set up the Gaussian surface for two situations using this long cylindrical volume, which in this case the Gaussian surface would also be a cylinder. The first surface is inside the cylinder, and that will affect the internal charge. This is when we go back to our ϱ = Q/V, because instead of Q=ϱV, we want Qinternal which = ϱVinternal. Since r <=R, the V internal is πr^2L, and Q internal is ϱπr^2L. Plug all these values in, and then isolate to get the E field. The second Gaussian surface outside the field is similar, but the Q internal in this case is just the Volume of the long cylinder, which has a radius notated by a capital R.

3)

its the "charge per unit length" so $\frac{Q}{L}$

$3 = \frac{Q}{L}$

$Q = 3L$ rearrange & plug in

a) E inside cylinder:

charge is on the surface, so there is no q_{in}, and thus $E=0$

E outside cylinder:

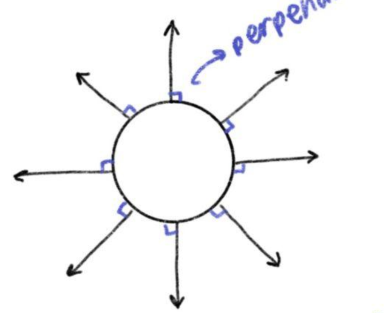
$\oint E\cdot dA = \frac{q_{in}}{\varepsilon_0}$ start w/ gauss's law

$E(2\pi r L) = \frac{3L}{\varepsilon_0}$ plug in $2\pi r L$ for surface area of a cylinder

$E = \frac{3L}{2\pi r L \varepsilon_0}$

$\boxed{E = \frac{3}{2\pi r \varepsilon_0}}$

b)



Efield points outward since the cylinder has a positive charge

Similar to the last problem, this problem also requires subbing in a expression for the q internal. As explained, there is no E field inside the cylinder as there is no net charge inside the cylinder. However, on the outside, there is a "charge per unit length" that we can rearrange to find that Q=3L. We can then plug this into our Gauss's Law formula as the q in and isolate to find the E field on the outside. For part B, the electric field lines are perpendicular to the rod as they point radially outward due to the positive charge of the cylinder.