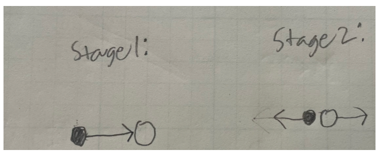
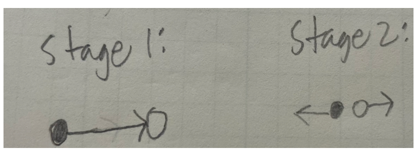
Beckett Hutchens
Chapter 9: Momentum
Background: Unit 9 explores how to calculate an object's momentum when acted upon by a force and the law of conservation of momentum.
Major topics:
Linear Momentum
Impulse
Conservation of Linear Momentum
Different types of collisions
Center of Mass
Important terms:
1. Linear Momentum: A quantity of motion that is the result of the product between an object's mass and velocity.
2. Impulse: The force applied over a given amount of time, resulting in a change in the momentum of an object.
3. Conservation of Linear momentum: The impulse that objects apply to one another is equal in magnitude and opposite in direction, so the combined momentum of all objects remains the same.

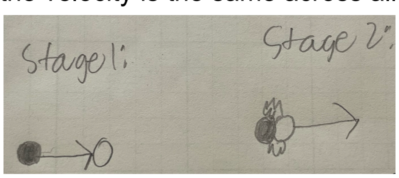


4. Types of collisions: (Can be in multiple dimensions)
- Elastic collision: Momentum is conserved and minimal to no kinetic energy is lost.

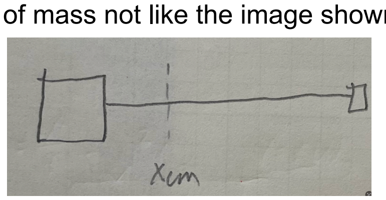


- Inelastic collision: Momentum is conserved and kinetic energy is lost due to heat.

- Perfectly inelastic collision: Inelastic collision where objects stick together, so mass is added and the velocity is the same across all objects.



5. Glancing collision: One mass strikes an identical stationary mass
6. Center of mass: The center of mass is the "weighted average position of the system's mass," so if a force is applied at this position, the mass will not rotate. An object can even have a continuous distribution of mass not like the image shown below.



7. Density: The amount of mass in either the linear, area, or volume.

Important Formulae:

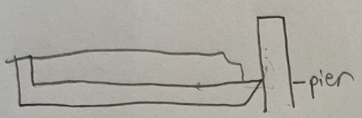
(ALL p's and v's should have the vector symbol above it)

Momentum (Vector)	p=mv, Unit: Newton-seconds
Momentum in terms of force	F=(dp)/(dt)
Impulse (Vector)	$J=\int_{ti}^{tf}dt=\Delta p=F\Delta t=m\Delta v$
Conservation of Linear Momentum	p1+p2=p1'+p2' m1v1+m2v2...+mnvn= m1v1'+m2v2'+ ... mnvn'
Elastic collision	p1+p2=p1'+p2' and k1+k2=k1'+k2'
Inelastic collision	p1+p2=p1'+p2'
Perfectly inelastic collision	p1+p2=p' m1v1+m2v2=(m1+m2)v'
Center of Mass	$Xcm=(\sum m_i x_i)/(\sum m_i)$ $Ycm=(\sum m_i y_i)/(\sum y_i)$ $Zcm=(\sum m_i z_i)/(\sum z_i)$ Rcm: $Xcm\hat{i} + Ycm\hat{j} + Zcmk$
Continuous distribution of mass	$Xcm=\lim_{\Delta mi->0} (\sum x_i m_i)/M= (1/M)\int xdm$
Density Equations	Volume: ϱ=(dm/dV)->dm=ϱdV Length: λ=(dm/dL)->dm=λdL Area: $\boldsymbol{\sigma}$=(dm/dA)->dm=$\boldsymbol{\sigma}$dA
System in motion	$r_{cm}=(\sum m_i r_i)/M$ $(d/dt)r_{cm}=(d/dt)(\sum m_i r_i)/(M)$ $v_{cm}=(\sum m_i v_i)/(M)$ $v_{cm}=(\sum p_i)/M$ $Mv_{cm}=\sum p_i$ $Mv_{cm}=p_{total}$

Practice Problems:

1. A cruise ship with a mass of $1.00x10^7$ kg strikes a pier at a speed of 0.750 m/s. It comes to rest after traveling 6.00 m, damaging the ship, the pier, and the tugboat captain’s finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest, assuming a constant force.)

First, draw the situation:



$m = 1.00 \times 10^7$ kg
$V = 0.750$ m/s
$x = 6.00$ m

Next, use the impulse equation:

$J = \Delta P$
$F \cdot t = P_f - P_i$, transform so that we are solving for F , plug in values

$F = \frac{mv_f - mv_i}{t} = \frac{1.00 \times 10^7 (0 - 0.750)}{t}$

but how do we find t?

we know V_{avg} and distance it takes to stop so...

$\Delta t = \frac{x}{\frac{V_i + V_f}{2}} = \frac{6m}{\frac{0.750 + 0}{2}} = 16s$

V_{avg}

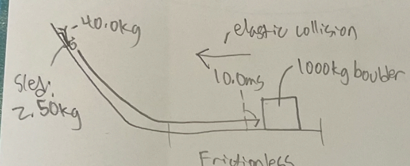
Plug Δt back into equation:

$F = \frac{1.00 \times 10^7 (0 - 0.750)}{16}$

$\boxed{F = 4.68 \times 10^5 N}$

2. A boy sleds down a hill and onto a frictionless ice-covered lake at 10.0 m/s. In the middle of the lake is a 1000-kg boulder. When the sled crashes into the boulder, he and the sled are propelled backwards from the boulder. The collision is an elastic collision. If the boy's mass is 40.0 kg and the sled's mass is 2.50 kg, what is the speed of the sled+boy and the boulder after the collision?
m1 and v1 are sled+boy, and m2 and v2 are boulder

First, draw the situation:



$m_1 = 40 + 2.5 = 42.5$ kg
$m_2 = 1000$ kg
$V_1 = 10.0$ m/s
$V_2 = 0$ m/s

For an elastic collision:
$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
and
$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

Now I will plug in values to the momentum equation: $1000 \cdot 0 = 0$

$42.5 \cdot 10 + 0 = 42.5v_1' + 1000v_2'$

$\frac{425 - 42.5v_1' = 1000v_2'}{1000}$, isolate one unknown on one side

$0.425 - 0.0425v_1' = v_2'$

Now I will plug into the energy equation:

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$, plug in values

$\frac{1}{2}42.5 \cdot 10^2 + 1000 \cdot 0^2 = 42.5v_1'^2 + 1000v_2'^2$

$4250 = 42.5v_1'^2 + 1000(0.425 - 0.0425v_1')^2$

$4250 = 42.5v_1'^2 + 1000(0.00181v_1'^2 - 0.0361v_1' + 0.181)$

$4250 = 36.1v_1' + 144.3v_1'^2 + 181$

$0 = 44.3v_1'^2 + 36.1v_1' - 4070$, set equal to zero to make quadratic equations

solutions:
$\boxed{9.19}$ or -10.0
don't worry about signs at the end, choose the one that makes sense, it will lose some speed

negative because propelled backwards

$\boxed{V_{sled} = -9.18 m/s}$

Substitute and solve

Now plug to solve for v_2' or speed of boulder:

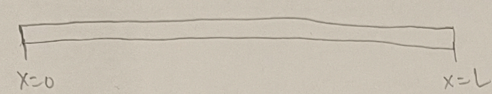
$0.425 - 0.0425 \cdot (-9.18) = v_2'$

$\boxed{V_{boulder} = 0.815 m/s}$

Note about solution, because we know some energy was transferred to the boulder, the final speed must be less than 10.0 m/s for the sled+boy.

3. What is the center of mass of a rod with changing density $\lambda = 1/4x^3$ from 0 to L?

First, draw the situation:



For continuous distribution of density:

$$X_{cm} = \frac{1}{M}\int x\,dm$$

in order to integrate in terms of x, must use $\lambda dx = dm$

$$\lambda dx = \frac{1}{4}x^3$$

$$X_{cm} = \frac{1}{M}\int \lambda x\,dx$$

– plug in λ and bounds

$$X_{cm} = \frac{1}{M}\int_0^L \frac{1}{4}x^4\,dx$$

– integrate

$$X_{cm} = \frac{1}{20M}\left. x^5 \right|_0^L$$

$$X_{cm} = \frac{L^5}{20M}$$

– Now, we still need to get rid of M

For the rod:

$$M = \int_0^L dm$$

we know $dm = \lambda dx$ so...

$$M = \int_0^L \frac{1}{4}x^3\,dx$$

$$M = \frac{1}{16}\left. x^4 \right|_0^L$$

$$M = \frac{L^4}{16}$$

plug back in

$$X_{cm} = \frac{L^5}{20\left(\frac{1}{16}L^4\right)}$$

$$\boxed{X_{cm} = \frac{4L}{5}}$$