Sharon Chou

Resources: crashwhite.com and openstax University Physics Volume 1 textbook

Chapter 7: Work & Kinetic Energy

Background/Summary:

This chapter is about force applied to an object as it is displaced (moves a certain distance with direction). It explores when that force is varying, when that force is applied by an ideal spring, and the time rate at which that force is applied.

Vocabulary:

- **Work:** a scalar quantity. Energy is the ability to do work & work is the transfer of energy
- **Equilibrium position:** in the context of a spring, the position at which the attached mass experiences no net force from the spring
- **Power:** the time rate at which work is done

Major Topics (Including Formulae & When to Use):

Formula	When to Use
Dot Product $= A \cdot B$ $C = (\vec{A}_x \cdot \vec{B}_x) + (\vec{A}_y \cdot \vec{B}_y)$ $= AB\cos\theta$	→ find the product of 2 vectors • find work $= \vec{F} \cdot \vec{\Delta X}$ (J) ↳ $\vec{A}$ = Force ↳ $\vec{B}$ = displacement (Δx) ↳ θ = angle between F & ΔX
$W = \int_{x_i}^{x_f} F(x)\, d\vec{x}$ $d\vec{x}$: over tiny displacement	→ force varies by position • given F(x) function ↳ integrate • given F(x) graph ↳ area under curve = work

If work is scalar, how can it have signs? The signs don't indicate direction. Rather, if work is negative, it is work that is done to make the object NOT move (force is decreasing the energy of the object). Vice versa, positive work is done to help the object move (force is increasing the energy of the object).

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Hoohe's Law:

$F_{spring} = -k\Delta X$ (N)

$k = \frac{-F_s}{\Delta x}$ (N/m)

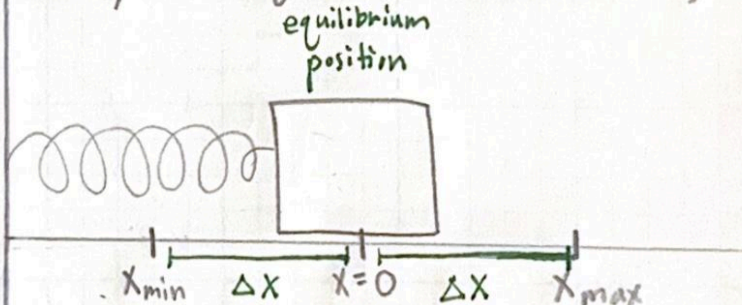
$W_{spring} = \frac{1}{2}kx^2$

↳ x relative to x=0 at equilibrium

→ ideal spring compressed or stretched. ⊖ = F_s opposite direction of displacement (if always add F_s then account for direction of Δx → right sign)

→ k: constant of proportionality

→ work done by spring in returning mass to equilibrium pos. (away from equilibrium → $-\frac{1}{2}kx^2$)



Work-Kinetic Energy Theorem:

$\sum W = K_f - K_i = \Delta K$

$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

↳ definition of kinetic energy

→ especially helpful when:

- force varies
- want to find velocity/ΔK

& can find work easily

→ could have potential energy, it doesn't impact

$P_{average} = \frac{Work}{\Delta time}$ (Watts)

→ > quickly work done, > power required

$P_{inst} = \frac{dW}{dt}$

↓

$P_{inst} = \vec{F} \cdot \vec{v}$

→ given W(t) equation

↳ take derivative

→ given / can find force & velocity

$P_{average} = \frac{\Delta U}{\Delta t} = \frac{mgh_f - mgh_i}{\Delta t}$ ← U_g (gravitational potential energy) = mgh

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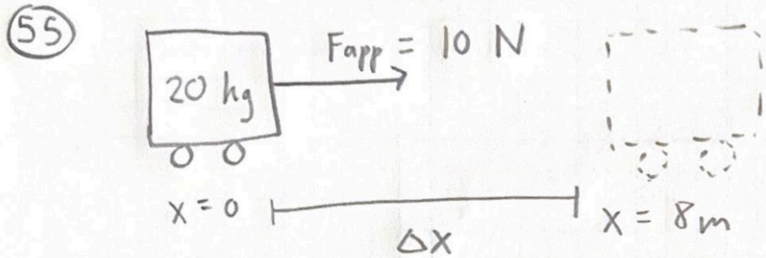
Problems in Order of Difficulty (Easy, Medium, Hard):

55. A constant 10-N horizontal force is applied to a 20-kg cart at rest on a level floor. If friction is negligible, what is the speed of the cart when it has been pushed 8.0 m?

55



$\Sigma W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ ← Work-Kinetic Energy Theorem

$W = \vec{F} \cdot \vec{x} = Fx\cos\theta$

$Fx\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$(10)(8)(\cos 0) = (\frac{1}{2})(20)[v_f^2 - 0^2]$

↳ parallel F & Δx

$\frac{80}{10} = v_f^2$

$v_f = \sqrt{8} = \boxed{2.83 \text{ m/s}}$

71. (a) What is the average useful power output of a person who does 6.00×10^6 J of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

71 (a) $P_{avg} = \frac{\text{Work}}{\Delta t}$

$P_{avg} = \frac{6e6 \text{ J}}{28800 \text{ s}}$ ← $8 \text{ hr} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{60 \text{ sec}}{1 \text{ min}}$

$= \boxed{208 \text{ W}}$

(b) Work = ?

$W = \vec{F} \cdot \vec{x}$ ← we'll find $\vec{F}$ using $F_g = mg$ because the $\vec{F}$ used to lift the bricks is working against the force of gravity

$W = mg\Delta x$ ↖ θ irrelevant, F & x are parallel

$W = (2000)(9.8)(1.5) = 2.94e4 \text{ J}$

$P_{avg} = \frac{\text{Work}}{\Delta t} \rightarrow \Delta t = \frac{\text{Work}}{P_{avg}}$

$\Delta t = \frac{2.94e4 \text{ J}}{208 \text{ W}} = \boxed{141 \text{ seconds}}$

41. Engineers desire to model the magnitude of the elastic force of a bungee cord using the equation

$$F(x) = a\left[\frac{x+9\text{ m}}{9\text{ m}} - \left(\frac{9\text{ m}}{x+9\text{ m}}\right)^2\right],$$

where *x* is the stretch of the cord along its length and *a* is a constant. If it takes 22.0 kJ of work to stretch the cord by 16.7 m, determine the value of the constant *a*.

(41) $W = \int_{x_i}^{x_f} F(x)\, d\vec{x}$

$22e3 = \int_0^{16.7} a\left[\frac{x+9}{9} - \left(\frac{9}{x+9}\right)^2\right] dx$

$22e3 = a\left[\int_0^{16.7} \frac{x+9}{9} dx - \int_0^{16.7} \left(\frac{9}{x+9}\right)^2 dx\right]$

↳ constants can be brought out & terms can be split into 2 integrals if they're added/subtracted

$\int_0^{16.7} \frac{x+9}{9} dx = \frac{1}{9}\int_0^{16.7} x+9\, dx = \frac{1}{9}\left[\int_0^{16.7} x\, dx + \int_0^{16.7} 9\, dx\right]$

$= \frac{1}{9}\left[\frac{1}{2}x^2\Big|_0^{16.7} + 9x\Big|_0^{16.7}\right] = 32.2$

MINUS

$\int_0^{16.7} \left(\frac{9}{x+9}\right)^2 dx = \int_0^{16.7} \frac{81}{(x+9)^2} dx = \int_0^{16.7} 81(x+9)^{-2} dx$

$= -81(x+9)^{-1}\Big|_0^{16.7} = 5.85$

$22e3 = a[32.2 - 5.85] = 26.35a$

$a = 22e3 \div 26.35 = \boxed{835 \text{ N}}$