Sharon Chou

Resources: crashwhite.com and openstax University Physics Volume 1 textbook

Chapter 13: Universal Gravitation

Background/Summary:

This chapter is about what happens to gravity and gravitational force when we're no longer looking at Earth's surface but much higher, into space, and sometimes between two celestial bodies. It also deals with potential energy and orbits in general.

Vocabulary:

- **Geosynchronous Orbit:** an orbit that matches Earth's rotation
- **Escape Velocity:** velocity needed to escape an object's gravity field
- **Elliptical Orbit:** an oval-shaped orbit
    - **Perihelion:** point at which orbiting object is closest to what it's orbiting around
    - **Aphelion:** point at which orbiting object is furthest to what it's orbiting around

Major Topics (Including Formulae & When to Use):

$\vec{F}_g = -G\frac{m_1 m_2}{r^2}\hat{r}$ G: gravitational constant: $6.674e{-11}\ \frac{N\cdot m^2}{kg^2}$ r^2: distance between 2 masses $\hat{r}$: direction, $-\hat{r}$ = pointing inward	• g is only 9.8 on surface of Earth – as get further from Earth, less acc. • universal vs. "normal" gravitation? universal → space; "normal" → on Earth, near surface * r in problems including Earth = relative to Earth's center – add Earth's radius (6.38e6 m) to any surface measurement
$U_g = -\frac{Gm_1 m_2}{r}$ • multiple masses; $U_{net} = \sum U_i$	• Gravitational potential • assumes $U_i = 0$ @ $r_i = \infty$ • our values are ⊖, not matter though because we care about ΔU

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$$E_{total} = \frac{1}{2}mv^2 + -\frac{GMm}{r}$$

for orbiting masses

v ↓ $v_{tangential}$, not ω

$$E_{bound} = \frac{1}{2}U = -\frac{1}{2}\frac{GMm}{r}$$

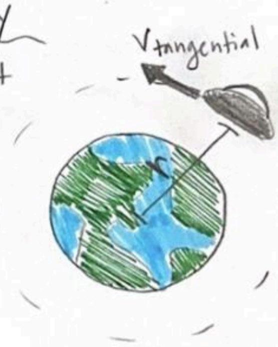
↓ object continues circular path without falling or flying away

Orbits & Energy

← radius ≠ need to be constant

← can be non-circular orbit

← derivation: $\sum E = K + U$



← circular paths only

← derivation:

$$F_c = F_g \Rightarrow \frac{mv^2}{r} = \frac{GMm}{r^2}$$

• multiply both sides by $\left(\frac{r}{2}\right)$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r} \quad \text{aha!} \quad K = -\frac{1}{2}U$$

$$\Downarrow$$

$$E_{total} = K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

• substitute in $K = -\frac{1}{2}U$

$$E_{total} = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

$$E_{total} = \frac{1}{2}U$$

Problems in Order of Difficulty (Easy, Medium, Hard):

21. (a) What is the acceleration due to gravity on the surface of the Moon? (b) On the surface of Mars? The mass of Mars is 6.418×10^{23} kg and its radius is 3.38×10^6 m.

(21) (a) a due to gravity on surface of moon = ?

$$F_{net} = ma$$

$$F_g = ma \quad \swarrow \quad |F_g| = \frac{Gm_1m_2}{r^2}$$

$$\frac{Gm_1\cancel{m_2}}{r^2} = \cancel{m_2}a$$

← where m_1 is the moon and m_2 is any object with an irrelevant mass (will cancel out)

$$\frac{Gm_1}{r^2} = a$$

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• from the astronomical data in textbook & NASA:
- mass of moon: 7.36e22 kg
- radius of moon: 1740 km $\cdot \frac{1000\ m}{1\ km}$ = 1,740,000 m

$$\frac{(6.674e{-}11)(7.36e22)}{(1.74e6)^2} = a \quad \leftarrow G = 6.674e{-}11,$$
gravitational constant

$$\boxed{a = 1.62\ m/s^2}$$

(b) $\frac{Gm_1}{r^2} = a$ ← same derivation as above

$$a = \frac{(6.674e{-}11)(6.418e23)}{(3.38e6)^2} = \boxed{3.75\ m/s^2}$$

31. What is the escape speed of a satellite located at the Moon's orbit about Earth? Assume the Moon is not nearby.

(31) $V_{escape} = ?$

• final position is infinitely far from Earth, K_f & $U_f = 0$

$$K_i + U_i = K_f + U_f$$ ← Conservation of Mechanical Energy

$$\frac{1}{2}mV_{esc}^2 + -\frac{Gm_1m_2}{r} = 0 + 0$$ ← m_1 = Earth, m_2 = the satellite

$$V_{esc} = \sqrt{\frac{2Gm_1}{r_i}}$$

$$V_{esc} = \sqrt{\frac{2(6.674e{-}11)(5.97e24)}{(384,400,000)}}$$ ← G = gravitation constant, mass of Earth (textbook), distance of moon from Earth (NASA)

$$\boxed{V_{esc} = 1440\ m/s}$$

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43. (a) In order to keep a small satellite from drifting into a nearby asteroid, it is placed in orbit with a period of 3.02 hours and radius of 2.0 km. What is the mass of the asteroid?

43 $F_c = F_g$

$\frac{m_2 v^2}{r} = \frac{G m_1 m_2}{r^2}$ ← m_2 = satellite, m_1 = asteroid

• find velocity:

– period: $3.02 \text{ hours} \cdot \frac{60 \text{ min}}{1 \text{ hour}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 10872$ seconds

– circumference: $2\pi r = 2\pi(2000) = 12566$ m

– velocity: $\frac{\Delta x}{\Delta t} = \frac{12566}{10872} = 1.16$ m/s

$v^2 = \frac{G m_1}{r}$

$v = \sqrt{\frac{G m_1}{r}}$

$1.16 = \sqrt{\frac{(6.674e-11)(m_1)}{2000}}$

$\boxed{m_1 = 4.03e13 \text{ kg}}$