Lab: AP Review Sheets Brandon Chui

Motion in Two Dimensions

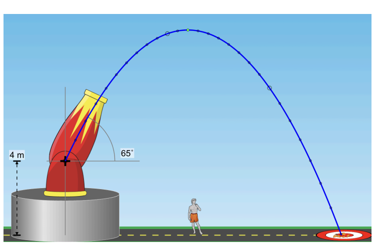
Background:
This chapter covers motion in two dimensions, which builds off of our understanding of vectors.

Major Topics:
The major topics covered in this chapter are projectile motion, uniform circular motion, and relative motion.

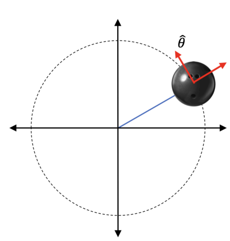
Important Vocabulary:
- Projectile Motion: an object moves through the air and is only subject to the force of gravity
- Uniform Circular Motion: motion of an object in a circular path
- Centripetal/Radial Acceleration: acceleration of an object moving in a circle that points toward the center of the circle
- Tangential Acceleration: acceleration of a changing speed, moving object that acts along the tangent to the curved path
- Unit Vectors for Circular Motion:
  - $\hat{\theta}$: unit vector tangent to the circular path (positive indicates counterclockwise)
  - $\hat{r}$: unit vector along the radius vector (positive indicates outwards)
- Relative motion: motions occurring relative to different frames of reference

Important formulae:
- Kinematics Equations (continuation from previous chapters):
  - $v = v_0 + at$
  - $\Delta y = v_0 t + \frac{1}{2}at^2$
  - $v^2 = v_0^2 + 2a\Delta x$
- Centripetal/Radial Acceleration: $a_c = \frac{v^2}{r}$
- Tangential Acceleration: $a_{tan} = \frac{dv}{dt}$
- Relative Motion: $\vec{v}_{a\ relative\ to\ b} + \vec{v}_{b\ relative\ to\ c} = \vec{v}_{a\ relative\ to\ c}$

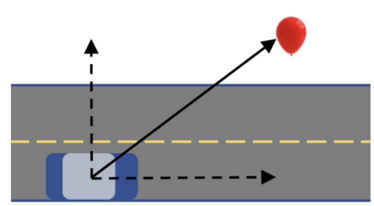
Diagrams:



Projectile Motion



Uniform Circular Motion



Relative Motion

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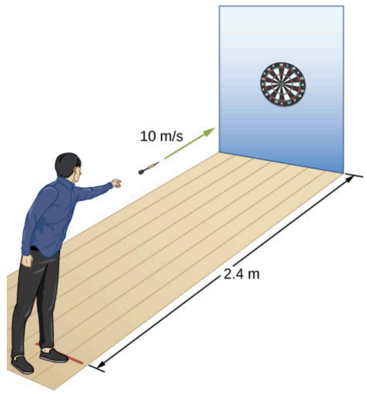
Motion in Two Dimensions

1. A dart is thrown horizontally at a speed of 10 m/s at the bull's-eye of a dartboard 2.4 m away, as shown in the following figure. How far below the intended target does the dart hit?



Solution:

To solve this projectile motion problem, we need to use our kinematics equations. We are given Δx and V_{x0}. To find Δy, the first step is finding t.

$$\Delta x = V_x t$$

$$t = \Delta x / V_x = 2.4/10 = 0.24s$$

Using t, we can now solve for Δy.

$$\Delta y = V_{y0} t + \frac{1}{2} a t^2$$

$$\Delta y = (0)(0.24) + \frac{1}{2}(9.80)(0.24)^2 = 0.28224 \rightarrow \boxed{0.282m}$$

Motion in Two Dimensions

2. A fan is rotating at a constant 360.0 rev/min. What is the magnitude of the acceleration of a point on one of its blades 10.0 cm from the axis of rotation?

Solution:

To solve this uniform circular motion problem, we need to convert 360.0 rev/min to m/s to find v.

$$360.0 \frac{rev}{min}\left(\frac{1\ min}{60\ s}\right) = 6.0\ rev/s$$

We need to calculate a_c 10.0 cm from the axis of rotation, and we can use that info to convert 6.0 rev/s to m/s.

$$C = 2\pi r = 2\pi(0.10) = 0.20\pi$$

$$6.0 \frac{rev}{s}\left(\frac{0.20\pi\ m}{1\ rev}\right) = 3.7699 \rightarrow 3.8\ m/s$$

Now we can plug in v to find a_c.

$$a_c = \frac{v^2}{r} = \frac{3.8^2}{0.10} = 144.4 \rightarrow \boxed{144\ m/s^2}$$

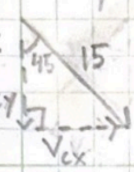
Motion in Two Dimensions

3. A cyclist traveling southeast along a road at 15 km/h feels a wind blowing from the southwest at 25 km/h. To a stationary observer, what are the speed and direction of the wind?

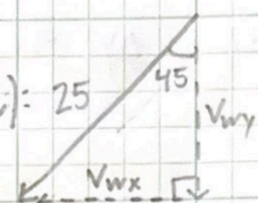
Solution:

To solve this relative motion problem, we first need to break the velocity vectors of the cyclist and the wind into their x and y components.

cyclist ($\vec{v}_c$):



wind ($\vec{v}_w$):



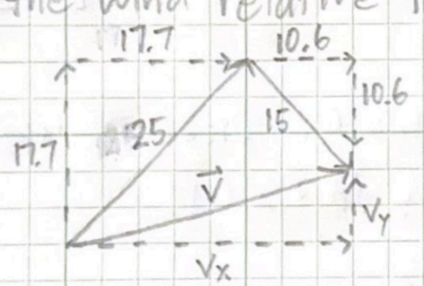
$v_{cy} = 15\cos 45 = 10.6 \text{ km/h}$

$v_{cx} = 15\sin 45 = 10.6 \text{ km/h}$

$v_{wy} = 25\cos 45 = 17.7 \text{ km/h}$

$v_{wx} = 25\sin 45 = 17.7 \text{ km/h}$

Now we need to find the magnitude and direction of the wind relative to a stationary observer.



$v_x = v_{wx} + v_{cx} = 17.7 + 10.6 = 28.3 \text{ km/h}$

$v_y = v_{wy} - v_{cy} = 17.7 - 10.6 = 7.1 \text{ km/h}$

$\vec{v} = \sqrt{28.3^2 + 7.1^2} = \boxed{29.2 \text{ km/h}}$

$\theta = \tan^{-1}(7.1/28.3) = \boxed{14.1° \text{ N of E}}$