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# Chapter 13: Universal Gravitation

**Background/Summary:**

In this unit, we will delve into different concepts surrounding gravity, orbits, gravitational potential energy, escape velocity, and more.

**Major Topics, Vocabulary, and Formulae**:

1. Newton's Law of Universal Gravitation

In the past, we have calculated the force of gravity only on Earth using the formula $F_g = mg$, but there is another formula for calculating Fg that takes into account the mass of the object ($m_1$), the mass of the body ($m_2$) exerting the force of gravity, the distance between the two objects ($r$), and the unit vector between the two ($\hat{r}$). We use this formula when the acceleration caused by gravity is not g.

Newton's Law of Universal Gravitation:

$\boxed{F_g = G\frac{m_1 m_2}{r^2}\hat{r}}$ where $G = 6.67 * 10^{-11} \frac{(N*m^2)}{kg^2}$

The "G" in this equation is the symbol for the universal gravitational constant, which, according to the equations sheet, equals. This law applies to all objects.

Note: when you are given the radius of an object from Earth, remember to include the radius of Earth ($r = 6.38 * 10^6 m$) in your total radius for the equation. We also sometimes call Earth's mass "M."

Fun fact: we can also use this equation to derive the acceleration of objects on earth (g) with the following derivation:

$$F_g = -\frac{GMm}{r}$$

$$\cancel{m}a_g = -\frac{GM\cancel{m}}{r}$$

plug in G and earth's mass and radius

$$a_g = -\frac{GM}{r} = \frac{(6.672e^{-11})(5.98e24)}{(6.38e6)^2}$$

$$= \boxed{-9.80 m/s^2}$$

Another way you may see this equation is in calculating geosynchronous orbit, which refers to the orbit of an object matching Earth's rotation.

$$F_g = F_{centripetal}$$

use Newton's Universal Law of Gravitation and $F_c = ma_c$, $F_c = m\frac{v^2}{r}$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

cancel masses and radius

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$ for geosynchronous orbit

2. <u>Gravitational Potential Energy</u>

Another concept covered within this unit is gravitational potential energy. In previous units, we used $\Delta U = U_f - U_i = -\Delta W = \int_{x_i}^{x_f} F_g * dx$. However, as highlighted in the previous section, we need to use Newton's Law of Universal Gravitation when calculating $F_g$, and we would also need to use radius, not x value. The derivation for this equation is as follows:

$$\Delta U = \int_{r_i}^{r_f} F_g \cdot dr$$

plug in $F_g = -\frac{GMm}{r^2}$

$$\Delta U = \int_{r_i}^{r_f} -\frac{GMm}{r^2} \cdot dr$$

pull the constants out of the integral

$$\Delta U = -GMm \int_{r_i}^{r_f} \frac{1}{r^2} dr$$

solve the integral

$$\Delta U = -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

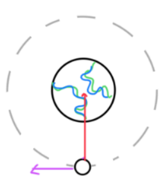
↳ $U_i = 0$, and $r_i = \infty$

$\boxed{U = -\frac{GMm}{r}}$, and when looking at a system with multiple masses,

$\boxed{U_{total} = \sum U}$

3. <u>Orbits and Energy</u>

Recall that the energy of a system is $E_{total} = U + K = \frac{-GMm}{r} + \frac{1}{2}mv^2$. Within this unit, we can consider problems where there is a satellite, which is a body in orbit around another, that is "bound" in a circular orbit with a centripetal force provided by gravity. We can calculate the energy in the system using the following analysis:



<u>Force analysis:</u>

$$F_c = F_g$$

$$\frac{r}{2} \cdot \frac{mv^2}{r} = \frac{GMm}{r^2} \cdot \frac{r}{2}$$

we want to find K, which is $\frac{1}{2}mv^2$, so we multiply both sides by $\frac{r}{2}$ to get K on the left

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$K = -\frac{1}{2}U$$

Back to the equation $E = U + K$, if we plug in $K = -\frac{1}{2}U$

$$E = U + K$$

$$E = U - \frac{1}{2}U = \boxed{\frac{1}{2}U} \text{ or } \boxed{-\frac{GMm}{2r}}$$

It makes sense that the energy of this system is negative because the gravitational potential energy of a system is negative when the radius < ∞.

Another context in which we could see energy analysis is when calculating the escape velocity of an object, meaning the velocity required for an object to completely leave earth's orbit.

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_{esc}^2 + \frac{-GMm}{r} = 0 + 0$$

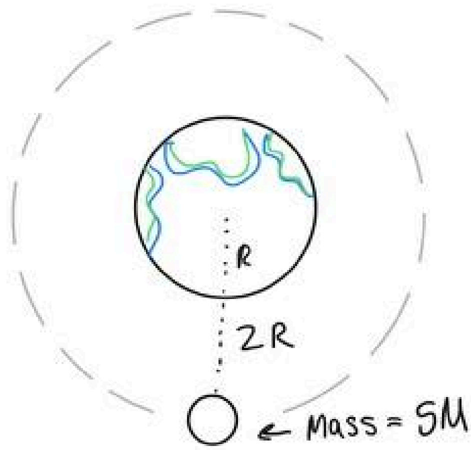
$$v_{esc} = \sqrt{\frac{2GM}{r_i}}$$

**Example Problems/FRQs**

1. Draw a graph of the gravitational potential energy between the Earth and the object as a function of the radius between the center of the Earth and the object, starting the graph with at the surface of the Earth. Provide a justification for this graph

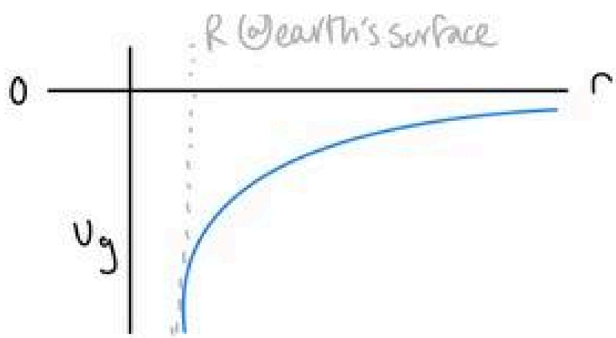
2. A satellite orbits the earth with a time period T=12 hours. What is the orbital radius?

3. With the values displayed in the diagram to the right, in terms of G, M, m, and R, find…
   a. Orbital speed
   b. Gravitational potential energy at r = 3R
   c. The escape velocity assuming that a similar object with mass 5M is launched from the surface of the earth



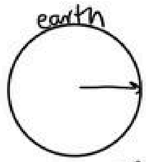

**Example Problems/FRQ Answers (Part I)**

**Problem 1 Solutions:**




From the equations sheet, $U_g = -\frac{GMm}{r}$,
so the radius from earth's surface is
inversly related to the $U_g$. As r increases
to $\infty$, the U approaches 0.

**Problem 2 Solutions:**




$m_{earth} = 5.84e24$

$r_{earth} = 6.37e6m$

$T = 12hrs \cdot \frac{3600s}{1hr} = \underline{4.32e4s}$

$v = \frac{d}{t} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{T}$

→ this leaves us to force comparison so we can relate v and r

$F_c = F_g$ ⤵ cancel m and r

$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$

$v^2 = \frac{GM}{r}$ ⤵ plug in $v = \frac{2\pi r}{T}$

$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$

$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$ ⤵ isolate r

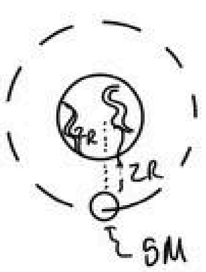
$r^3 = \frac{GMT^2}{4\pi^2}$

$r = \sqrt[3]{\frac{(6.672e{-11})(5.84e24)(4.32e4)^2}{4\pi^2}}$

$\boxed{r = 2.64e7m}$

↳ yay! not orbiting inside the earth!

**Problem 3 Solutions**:




a) Finding $v_{orbital}$

$F_g = F_c$

$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$ ⤵ plug in $R = R + 2R$

$\frac{GM}{2R} = v^2 \rightarrow \boxed{v = \sqrt{\frac{GM}{3R}}}$

b) Gravitational potential energy

$U = -\frac{GMm}{R}$ ⤵ plug in values

$= \boxed{-\frac{5}{3}\frac{GMm}{R}}$

c) escape velocity

$K_i + U_i = K_f + U_f$ ⤵ $v_f = 0$ when the rocket = out of earth's gravity as r increases, $U_f$ approaches 0

$\frac{1}{2}mv_i^2 + -\frac{GMm}{r} = 0 + 0$ ⤵ get v on one side and cancel masses

$\frac{1}{2}\cancel{m}v_i^2 = \frac{GM\cancel{m}}{2R}$

$\frac{1}{2}v_i^2 = \frac{GM}{2R}$ ⤵ isolate v

$\boxed{v_i = \sqrt{\frac{GM}{R}}}$