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Chapter 6: Circular Motion and More Forces

Background/Summary:

In this unit, we will further discuss Newton's Laws of Motion, specifically Newton's Second Law of Motion (F_{net}= ma), new contexts.

Major Topics, Vocabulary, and Formulae:

1. Friction

Friction: type of force that resists the relative motion of an object.
Within the context of this class, we will focus on sliding friction, which refers to the resultant force of friction due to two surfaces being in contact with each other. Friction depends on the $Force_{normal}$ as well as the coefficient of friction (μ), which is an experimentally determined, always positive value (between zero and one) that describes the "stickiness" between two objects. The greater the coefficient of friction, the greater the force of friction. We can calculate $F_{friction}$ by using the relationship between F_{normal} and μ.

$$Force_{friction} = Force_{normal} * \mu$$
$$[Newtons] = [Newtons][coefficient]$$

In this class, we focus on two types of sliding friction: static and kinetic.
Static friction: the force of friction between two surfaces when the two surfaces are not moving relative to each other; we denote the coefficient of friction for static situations as μ_s. In static situations, the formula above calculates the maximum possible static friction, not the actual static friction.

$$F_{static} < F_{normal}\,\mu_s$$

Kinetic friction: the force of friction between two surfaces when the two surfaces are moving relative to each other; we denote the coefficient of friction for kinetic situations as μ_k. The force of kinetic friction is less than the force of static friction because the rough surfaces of two objects are moving and no longer locked into place.

$$F_k = F_{normal}\mu_k \quad \text{thus,} \quad \mu_k < \mu_s.$$

2. Centripetal Force and Horizontal Circular Motion

Centripetal force: a type of force that produces centripetal acceleration, which is a type of center-seeking acceleration. Different types of forces can be considered centripetal depending on the circumstances. We describe this force using the following formulae:

$F_c = m\frac{v^2}{r}$ where m is the mass, v is the velocity, and r is the radius from the axis of rotation. In addition, $\Sigma F_c = \Sigma m\frac{v^2}{r}$.

Centripetal force and centripetal acceleration will be highlighted in problems concerning horizontal circular motion (ex. ball on a string, car on a turn, car at an incline, etc.)

3. Non-Uniform Circular Motion

Similar to how we can split up acceleration into x and y components and use i,j,k notation, we can also split up centripetal acceleration into a unit vector format with "hat theta" ($\hat{\theta}$) and "hat r" ($\hat{r}$), referring to the tangential acceleration and the radial acceleration, which are due to tangential and radial forces respectively. The following formulae explain the relationship between the different accelerations and forces:

$$a_{net} = a_{tangential} + a_{radial} \quad \& \quad F_{net} = F_{tangential} + F_{radial}$$

For example, $a_{net} = (0.5\,\hat{\theta} + 0.6\,\hat{r})\,m/s^2$ or $F_{net} = (0.7\hat{\theta} + 0.8\hat{r})\,N$

This kind of notation comes in handy in situations with non-uniform circular motion, such as a pendulum, especially if you need to take gravity into account when calculating the centripetal acceleration, etc.

brief note: centrifugal force is not a thing – people use the term centrifugal force to describe a force pulling an object away from the axis of rotation/center of a circle, but there is no such thing – an object being pulled from a circle can be caused by many things (ex. friction, inertia, etc.), but not an "apparent" force like centrifugal force

4. Resistive (Drag) Forces:

Drag force: a type of force that acts in resistance to a body's movement through a fluid substance (gas or liquid). Within this course, we will focus on air friction, which we also refer to as "drag." We can use the following formulae to calculate drag in two different circumstances:

Drag proportional to velocity:

$R = -bv$

where b is a constant

Drag proportional to velocity-squared:

$R = \frac{1}{2} D \rho A v^2$

where

$D =$ an experimentally-determined drag coefficient

$\rho =$ the density of the fluid

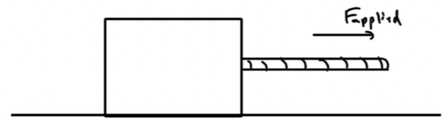
$A =$ the cross-sectional area of the moving object

*images above from crashwhite

When drag force is equal to the force of gravity for a falling object, that object no longer accelerates and reaches what is called terminal velocity.

Example Problems/FRQs

1. Sliding Box Part I: If there is a 5 kg container being pulled by a 40 N force along a table with a coefficient of kinetic friction of 0.6.
    a. What is the force of friction on from the container onto the table?
    b. What is the horizontal acceleration of the container?

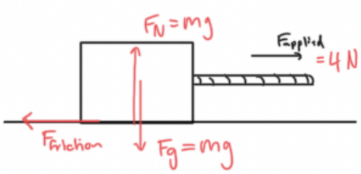


2. Pendulum: If there is a pendulum with a mass (20kg) hanging from a string with a radius of 60m released at $\theta = 60^\circ$ frowith an initial velocity of 0 m/s, find the force of tension at the Y_{min} and Y_{max}.

3. Sliding Box: Part II (the next level): The same 5kg container receives an initial push by a force applied so it slides with an velocity $v_0 = 40$ m/s at time t=0 and begins just sliding along a new frictionless surface with a b value of 5. Solve for the velocity after the box experiences drag for 5 seconds, assuming the only friction the box experiences is drag. Should the box speed up or slow down, and does this match your answer?

Example Problems/FRQ Answers

Problem 1 Solutions:



a) → We can use the formula $F_k = F_N \mu_k$

$F_k = F_N \mu_k$

$= (mg)\mu_k$

$= (5)(9.8)(0.6) = \boxed{29.4\,N}$

b) Using force comparison

$F_{net} = ma$

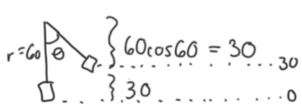
$F_{applied} - F_{friction} = ma$

$40 - 29.4 = 5 \cdot a$

$a = \frac{40 - 29.4}{5} = \boxed{2.12\ m/s}$

Problem 2 Solutions:

Before we get into force comparison, we need to find the velocity of the box as it swang from y_{max} to y_{min}. Assuming energy was conserved and there was no friction, which is a tad unrealistic, we can use conservation of energy to find the velocity @ y_{min}.



$K_f = U_i$ } expand

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g\Delta h$ } masses cancel

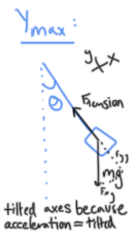
$\frac{v^2}{2} = g\Delta h$ } isolate v, $\Delta h = y_{max} - y_{min}$

$v = \sqrt{2g(y_{max} - y_{min})}$ } plugging in $y_{max} = 30m$, $y_{min} = 0m$

$v = \sqrt{2(9.8)(30 - 0)} = \underline{24.2\ m/s}$ @ y_{min} → this is also called the radial velocity;

For problem we only need radial forces because tension is a radial component, not tangential. We can do the same process to find the force of tension @ y_{max} and y_{min};

Problem 2 Solutions (continued)



Overall (Ymin and Ymax):

$F_{net\text{-}radial} = ma_{radial}$ ↰ using the centripetal acceleration for F=ma

$F_{Tension} - F_{g\text{-}radial} = m\frac{v^2}{r}$ ↰ $F_{g\text{-}radial} = mg\cos\theta$

$F_{tension} - mg\cos\theta = m\frac{v^2}{r}$

$F_{tension} = mg\cos\theta + m\frac{v^2}{r}$

↳ Now that we have the general equation for F_T in this situation, we can plug in numbers for Ymax and Ymin to find the F_T at each point.

Ymax:

$F_T = mg\cos\theta + m\frac{v^2}{r}$ ↰ v=0 at the instant when the box attatched to the wire are released → from the wire that pulls the box upward

$= (20)(9.8)\cos(60) + 0$

$= \boxed{98N}$

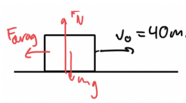
Ymin:

$F_T = mg\cos(0) + \frac{mv^2}{r}$

$= (20)(9.8) + \frac{(20)(24.2)^2}{(60)}$

$= \boxed{391 N}$

Problem 3 Solutions



We need to derive an equation for the velocity using the m, v_0, b, and t.

→ We know that drag $R = -bv$, and the only force acting on the box that would cause the motion to slow is the drag R.

$R = ma$ ↰ $R = -bv$

$-bv = ma$

$-bv = m\frac{dv}{dt}$ ↰ we want to set up a differential to solve for v, so we isolate "v" on one side

$-\frac{b}{m}dt = \frac{1}{v}dv$ ↰ set up boundaries

$\int_0^t -\frac{b}{m}dt = \int_{v_0}^{v_f} \frac{1}{v}dv$

$-\frac{b}{m}t = \ln|v_f| - \ln|v_0|$

$-\frac{b}{m}t = \ln\left|\frac{v_f}{v_0}\right|$ ↰ raise both sides e^{-} to get the velocities out of the ln( ).

$e^{-\frac{b}{m}t} = \frac{v_f}{v_0}$

$$\underline{\underline{v_f = v_0 e^{-\frac{b}{m}t}}}$$

Now, we can plug in the known values for v_0, b, m, and t

$v_f = (40)e^{-\frac{5}{5}\cdot 5} = \boxed{0.270 m/s}$

The box slowed down. This is expected because the box experiences a drag force opposing the motion and no longer has the applied force to push it forward. My answer reflects this concept.