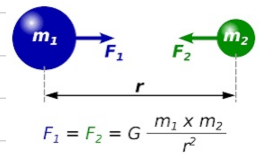
Chapter 13: Universal Gravitation

• Newton's Law of Gravitation Equation:

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}}$$



Where:

- $G \rightarrow$ (universal gravitation constant) 6.67e-11 $N \cdot m^2/kg^2$
- $m_1 \rightarrow$ mass of first object
- $m_2 \rightarrow$ mass of second object
- $r \rightarrow$ distance between the two

• Potential Gravitational Energy (+ derivation):

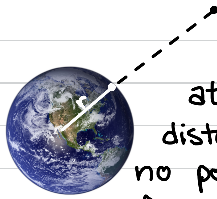
$$\Delta U = -\int_{r_i}^{r_f} F_g \cdot dr$$

$$= -\int_{r_i}^{r} -\frac{GMm}{r^2}\, dr$$

$$\Delta U = -GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$ goes to zero $/\frac{1}{\infty}$

$U_i = 0$ @ $r_i = \infty$

$$\boxed{U = -\frac{GMm}{r}}$$



$U_i = 0$
$r = \infty$

at a far away distance ($r = \infty$) there's no potential energy from Earth

☆ use conservation of energy U_i & K to find escape velocity → the min. speed required to leave a planet's gravitational force ☆

• Circular Orbit (+ energy derivation):

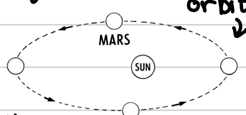
– the satellite (object circling a planet) stays on its path w/ centripetal force

$$F_c = \frac{mv^2}{r} \quad \& \quad F_g = \frac{GMm}{r^2}$$

$$\left(\frac{\cancel{r}}{2}\right)\frac{mv^2}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}\left(\frac{\cancel{r}}{2}\right)$$

$$K\left[\frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}\right.$$ //note $K = \frac{1}{2}U$

for elliptical orbits



use angular momentu
$(L = r \times mv)$

$E_{total} = K + U$ // use mechanical energy analysis

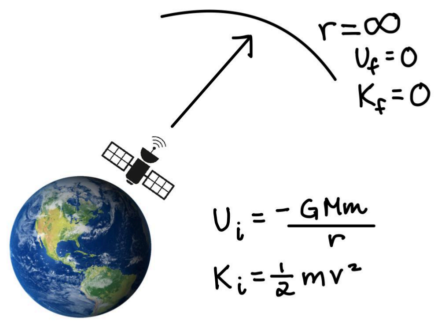
$$= \frac{1}{2}mv^2 - \frac{GMm}{r}$$

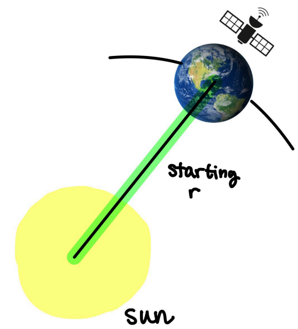
$$= \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r}$$

$$\boxed{E_{total} = -\frac{1}{2}\frac{GMm}{r} = \frac{1}{2}U}$$ in circular orbit

(#13 from Openstax Textbook) The International Space Station (ISS) has a mass of approximately 370,000 kg. (a) What is the force on a 150 kg suited astronaut if she is 20 m from the center of mass of the station? How accurate do you think your answer would be?

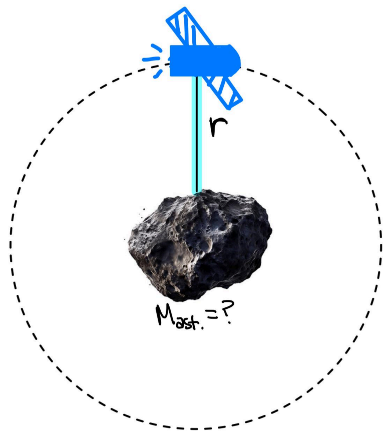
Work	Explanation
a) $m_2 = 150\ \text{kg}$ $M_1 = 370{,}000\ \text{kg}$ $r = 20\ \text{m}$ $F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67e{-}11)(150\ \text{kg})(370000)}{(20)^2}$ $= \boxed{9.25e{-}6\ \text{N}}$	Using Newton's Law of Universal Gravitation, simply plug in the mass of the ISS, the astronaut, and the radius between the two and solve.
b) I imagine that these values are very accurate because the space station isn't symmetrical, so this method doesn't hold.	N/A





(Example 13.7 on OpenStax) What is the escape speed from the surface of the Earth? Assume there is no energy loss from air resistance. Compare this to the escape speed from the Sun, starting from Earth's orbit.

Work	Explanation
a) $U_i + K_i = 0 + 0$ $\frac{1}{2}mv_i^2 + \frac{-GMm}{r} = 0$ $\frac{1}{2}\cancel{m}v_i^2 = \frac{GM\cancel{m}}{r}$ $\sqrt{v_i^2} = \sqrt{\frac{2GM}{r}}$ $v_i = \sqrt{\frac{2GM_{earth}}{r_{earth}}}$ $v_i = \sqrt{\frac{2(6.67e{-11})(5.96e24)}{(6.37e6)}}$ $= 1.12e4 \text{ m/s}$	The escape velocity is the minimum velocity needed in order to have no kinetic energy left (K final = 0). For our calculations, we are considering our final kinetic and potential energy to be zero and not affected by the gravitational effect of the Earth. Using conservation of energy analysis, we can use the kinetic energy equation and potential gravitational energy and isolate v. Once v is isolated, then plug in the given values.
b) $v_{\text{escape from sun}} = \sqrt{\frac{2GM_{sun}}{r_{sun-earth}}}$ $= \sqrt{\frac{2(6.67e{-11})(1.99e30 \text{ kg})}{(1.50e11 \text{ m})}}$ $= 4.21e4 \text{ m/s}$ The escape velocity from the sun is about 4x times as large as the escape velocity from the Earth.	Using the same equation above, we instead can use the mass of the sun and because the problem stated we are starting at Earth's orbit, using the radius of the orbit of the Sun and the Earth.



(#43 on Openstax) In order to keep a small satellite from drifting into a nearby asteroid, it is placed in orbit with a period of 3.02 hours and radius of 2.0 km. What is the mass of the asteroid?

Work	Explanation
#43) $T = 3.02\ \cancel{hr} \cdot \frac{3600\,s}{1\,\cancel{hr}} = 10872\,s$ $r = 2.0e3\ m$ $V = \frac{\Delta x}{t} = \frac{2\pi r}{T} = \frac{2\pi(2.0e3)}{10,872\,s}$ a) $F_c = F_g$ $\left(\frac{r^2}{G}\right)\frac{\cancel{m}v^2}{\cancel{r}} = \frac{\cancel{G}M\cancel{m}}{\cancel{r^2}}\left(\frac{r^2}{G}\right)$ $M_{asteroid} = \frac{rv^2}{G} = \frac{(2.0e3)\left(\frac{2\pi(2.03e3)}{10,872}\right)^2}{6.67e-11}$ $= \boxed{4.01e13\ kg}$	For an orbit, the centripetal force is equal to the force between the satellite and the asteroid. Isolate the “M” or the mass of the asteroid. To find the velocity, set it equal to the change in position over the period.The displacement is the circumference of the orbit and we have to convert the period from hours to seconds. Plug that value into the newly isolated M equation along with the radius of the orbit and the Universal Gravitation constant values and solve.