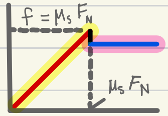
Chapter 6: Circular Motion

Summary: This guide will cover centripetal force, the force that keeps objects moving a circle, along with Drag & a brief review of friction

- **Friction** → a force that opposes the relative motion of a body
  - μ_k → for kinetic / sliding situations
  - μ_s → for static / non-sliding situations

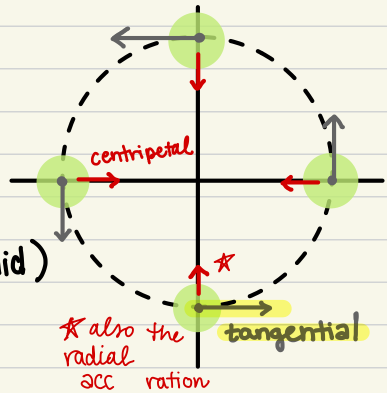


x-axis = $F_{applied}$
y-axis = f (friction force)

- **Centripetal Force** → a force that causes an object to accelerate centripetally; $F_{tension}$, $F_{applied}$, F_g, F_N, can act as a centripetal force

Equation:

$$\boxed{\begin{array}{ll} \Sigma F_c = m a_c & \\ \Sigma F_c = m\frac{v^2}{r} & a_c = \frac{v^2}{r} \end{array}}$$



- Resistive Force (R) varies according to the velocity through the "fluid" (gas or liquid)

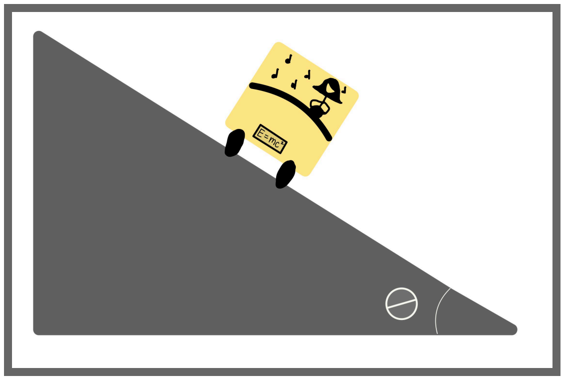
$R \propto v$	$R \propto v^2$
$R = -bv$	$R = \frac{1}{2} D \rho A v^2$
where:	where:
b = constant	D = drag coefficient
	ρ = density of the "fluid"
	A = cross-sectional area of object

Solving Strategy for Circular Motion:

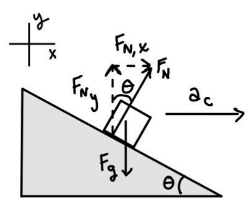
1) Draw a FBD

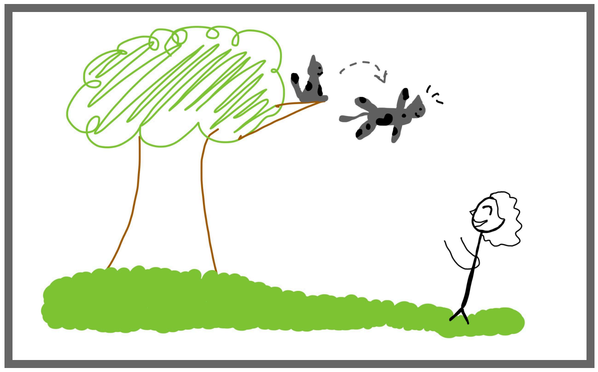
2) Determine which force is acting as the centripetal force

3) Set that force equal to the centripetal force equation & solve!



(#67 on Openstax) What is the ideal banking angle for a gentle turn of 1.2-km radius on a highway with a 105 km/h speed limit (about 65 mph), assuming everyone travels at the limit.

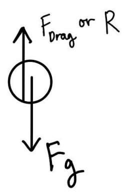
Work	Explanation
Know: $1.20\,\cancel{km}\cdot\frac{10^3\,m}{1\,\cancel{km}} = 1200\ m$ (radius) $\frac{105\,\cancel{km}}{\cancel{hr}}\cdot\frac{1\,\cancel{hr}}{3600\,s}\cdot\frac{10^3\,m}{1\,\cancel{km}} = 29.2\ m/s$ (speed)  $\Sigma F_x = ma$ $\Sigma F_y = ma_y$ $F_{N,x} = ma_c$ $F_{N,y} - F_g = 0$ $F_{N,x} = \frac{mv^2}{r}$ $F_{N,y} = mg$ $F_N \sin\theta = \frac{mv^2}{r}$ $F_N\cos\theta = mg$ $F_N = \frac{mg}{\cos\theta}$ $\frac{\cancel{m}g}{\cos\theta}\sin\theta = \frac{\cancel{m}v^2}{r}$ $g\tan\theta = \frac{v^2}{r}$ $\tan\theta = \frac{v^2}{gr}$ $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{(29.2\ m/s)^2}{(1200)(9.8)}\right) = \boxed{4.14°}$	After drawing a FBD, we decide **NOT** to turn the axis because although this is a banked turn, the centripetal acceleration isn't titled. Therefore, the **normal force must be broken into x and y components**. The forces in the y-direction include the y component for the normal force and the force of gravity. We can isolate and expand the normal force to get it in terms of mass, gravity, and cos of the angle. For the x-direction, the only force contributing is the x-component of the normal force, which is equal to the centripetal force. After expanding that component, we then have two equations. We can substitute the value of the normal force calculated from the y-direction analysis. Then, we make sine/cosine turn into tangent, divide the force of gravity to the other side, take the inverse of tan, and plug in the values to finish.



Your kitten has climbed a 5.0 m tall tree and refuses to come down. After bribing her with treats, she leaps off the tree. Blueberry weighs 850-g and has a surface area of 1130 cm². (a) Estimate its terminal velocity (Assume the drag coefficient, D is **0.70** and air density **ρ** is 1.293 kg/m³).

<table>
<tr><th>Work</th><th>Explanation</th></tr>
<tr><td>

a:



@ terminal velocity:

$F_{net} = ma = 0$

$R - F_g = 0$

$\frac{1}{2}D\rho Av^2 - mg = 0$

$\frac{1}{2}D\rho Av^2 = mg$

$v = \sqrt{\frac{2mg}{D\rho A}}$

$= \sqrt{\frac{2(0.850\,kg)(9.8\,m/s^2)}{(0.70)(1.293\,kg/m^3)(0.113\,m^2)}}$

$= \boxed{12.8\,m/s}$

$930\cancel{cm^2} \cdot \frac{1m^2}{(100cm)^2} = 0.093m^2$

know:

$m = 850g = 0.850kg$

Surface Area $= 1130\,\cancel{cm^2} \cdot \frac{1m^2}{(100\cancel{cm})^2} = 0.113\,m^2$

$D = 0.70$

$\rho = 1.293$

</td><td>At terminal velocity, the force of drag and the force of gravity are equal to one another because the object is no longer accelerating. Given the mass and area, we use the R equation proportional to velocity squared and subtract that with the force of gravity. To isolate v, we multiply by two, divide the Drag coefficient, air density, and cross-sectional area to the other side. Take the square of those terms and plug in the values.</td></tr>
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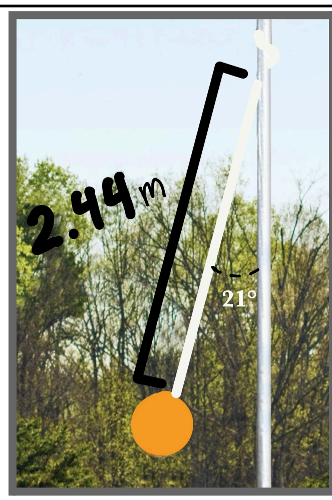

A 0.15 kg tetherball is suspended by a 2.44 m long string from the top of the pole and pulled back at an angle of 21°. The ball is then released so that it swings back and forth. The ball is traveling at 4.5 m/s at this given moment in time. (a) Draw a free-body diagram of the ball at this position, (INCLUDE COMPONENTS). (b) Calculate the force of tension in the wire at this moment. (c) Calculate the tangential and radial acceleration of the ball at this moment.

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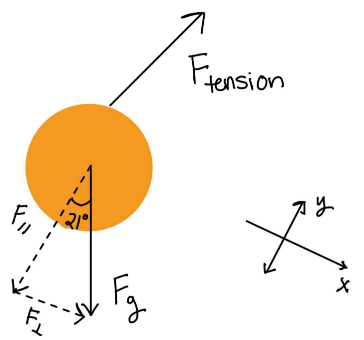

</td><td>The only two forces acting on the ball are the force of gravity and the force of tension from the string. Because the direction of motion of the ball is at an angle, we must tilt our axis and break up the force of gravity into components.</td></tr>
<tr><td>
$F_{net\,(radial)} = ma$ & $F_c = \frac{mv^2}{r}$
$F_{tension} - F_{\parallel} = \frac{mv^2}{r}$
$F_{tension} = \frac{mv^2}{r} + mg\cos(21)$
$= \frac{(0.15)(4.5)^2}{2.44} + (0.15)(9.8)\cos(21)$
$= \boxed{2.62\text{ N}}$
</td><td>The radial/centripetal acceleration accounts for the forces in the y direction. Therefore, the centripetal force is equal to the difference between the force of tension and the parallel component of the force of gravity. To solve for the force of tension, simply subtract gravity to the other side and plug in the values</td></tr>
<tr><td>
$F_{tangential} = ma_{tangential}$
$F_{\perp} = ma_{tangential}$
$a_{tangential} = \frac{F_{\perp}}{m}$
$= \frac{\cancel{mg}\sin(21)}{\cancel{m}} = g\sin(21)$
$= (9.8)\sin(21)$
$= \boxed{3.5\text{ m/s}^2}$

$F_{radial} = F_c = ma_c$
$a_c = \frac{v^2}{r} = \frac{(4.5)^2}{(2.44)} = \boxed{8.30\text{ m/s}^2}$
</td><td>The radial acceleration in this case is equal to the centripetal acceleration, so you use that equation and plug in the values. The tangential force accounts for the forces in the x direction, which, in this case, is only the perpendicular aspect of the force of gravity. To solve, divide the tangential force (mg sin(21)) by the mass of the ball.</td></tr>
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