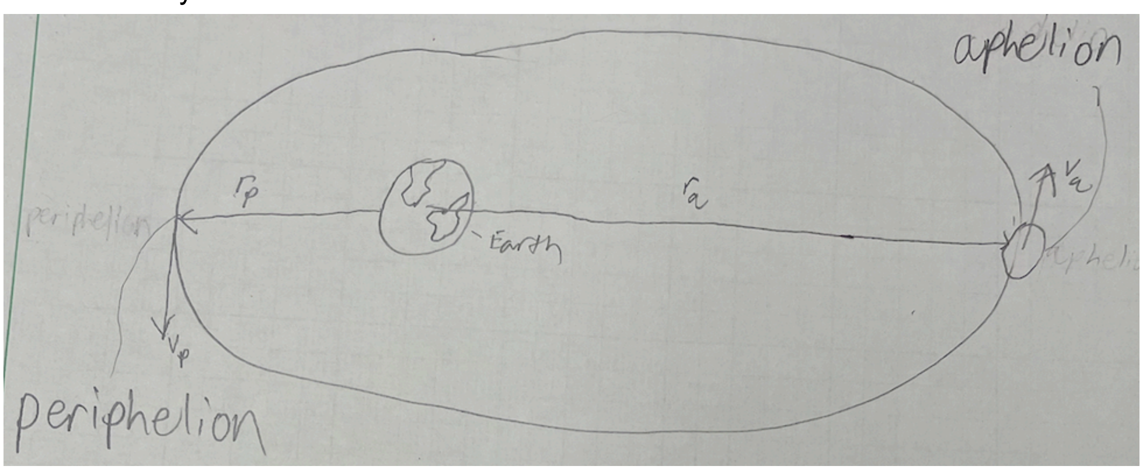
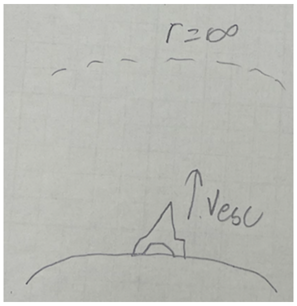
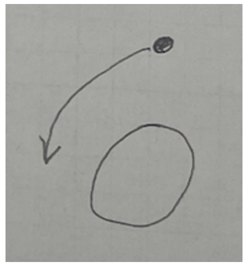
Beckett Hutchens
Chapter 13: Universal Gravitation
Background: In chapter 13, we will look at how the force of gravity influences a multitude of of concepts such as an object's acceleration, orbit, and energy.
Important topics:
Newton's Law of Universal Gravitation
Escape Velocity
Orbit
Gravitational Potential Energy
Energy of a Planet-Satellite System
Important Terms:
Newton's Law of Universal Gravitation:
Every particle has its own attractive force on other particles that is a product of the objects' masses and inversely proprotional to the square of the distance between the objects.
Universal Gravitation Constant: The G value that can relate the force of gravity to Newton's Law of Universal Gravitation.
Elliptical Orbits:

- Perihelion: The point of the shortest radius of the object to the body with the highest velocity
- Aphelion: The point of the longest radius of the object to the body with the lowest velocity



Gravity force: The force that a mass feels due to the presence of another mass nearby.
Gravity field: A field created around a mass that will produce a force on any object in the field.
Gravitational Potential energy: Energy that is created due to the force of gravity. This is calculated with different equations based on if an object is near the surface of earth or not.
Escape velocity: The velocity that an object needs to completely escape the gravity field of another object when launched from the surface of the other object into its orbit.



↑  ↓

Satellite: Any body that is in orbit around another.

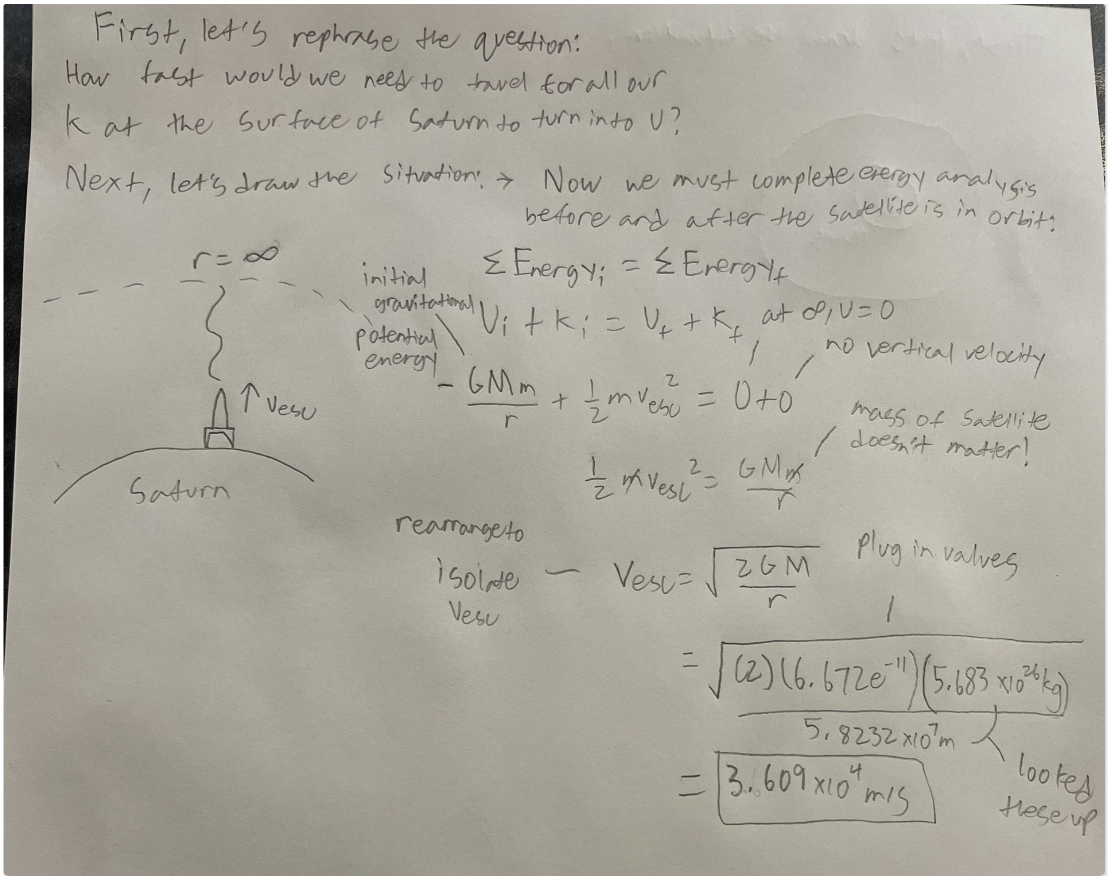
Important Formulae:

Name	Formula
Newton's Law of Universal Gravitation	$Fg=(Gm_1m_2)/(r^2)$
Universal Gravitation Constant	$G= 6.674 \times 10^{-11}$ N*(m^2)/(kg)
Gravitational Potential Energy	$W_{gravity}=U_i-U_f=-\Delta U$ $\Delta U=U_f-U_i=-W_{gravity}=-{}_{xi}\int^{xf}F_g*dx$
Potential Energy for a Mass in a Gravity Field where the earth is one of the masses	$U_f-U_i=-{}_{ri}\int^{rf}F_g*dr$ $U_f-U_i=-{}_{ri}\int^{rf}F_g*-(GM_{earth}m)/(r^2)*dr$ $U_f-U_i=-GM_{earth}m(1/r_f-1/r_i)$ Let $U_i=0$ at $r_i=\infty$ $U=-(GM_{earth}m)/r$
Gravitational Potential energy for multiple masses	$U_{total}=\sum U_i$ For three masses: $U_{total}=-(Gm_1m_2)/(r_{1,2})+(-Gm_2m_3)/(r_{2,3})+(-Gm_1m_3)/(r_{1,3})$
Energy of a Planet-Satellite System	$E_{total}=U+K$ $E_{total}=(-GMm)/(r)+(1/2)(mv^2)$
Energy of a bound, circular orbit	Using Force Analysis: $F_c=F_g$ $(mv^2)/(r)=(GMm)/(r^2)$ $(r/2))(mv^2)/(r)=(GMm)/(r^2)(r/2)$ $(1/2)mv^2=(1/2)(GMm)/(r)$ Note $K=-(1/2)U$ From what we developed in the Energy of a Planet-Satellite System: $E_{total}=U+K$ $E_{total}=(1/2)mv^2-(GMm)/(r)$ Because $(1/2)mv^2=(1/2)(GMm)/(r)$ $E_{total}=(1/2)(GMm)/(r)-(GMm)/(r)$ so $E_{total}=-(1/2)(GMm)/(r)$ $E_{total}=(1/2)U$, where $U=-(GMm)/r$ Therefore, the total energy for a satellite orbiting a mass is negative.

Problems:

1. What is the escape velocity from Saturn neglecting friction?

For Saturn: $M=5.683\times10^{26}$kg $r=5.8232\times10^{7}$ m



First, let's rephrase the question:
How fast would we need to travel for all our K at the surface of Saturn to turn into U?

Next, let's draw the situation: → Now we must complete energy analysis before and after the satellite is in orbit:

$r=\infty$

v_{esc}

Saturn

$$\Sigma \text{Energy}_i = \Sigma \text{Energy}_f$$

initial gravitational potential energy

$$U_i + K_i = U_f + K_f$$

at ∞, $U=0$

no vertical velocity

$$-\frac{GMm}{r} + \frac{1}{2}mv_{esc}^2 = 0+0$$

mass of satellite doesn't matter!

$$\frac{1}{2}\cancel{m}v_{esc}^2 = \frac{GM\cancel{m}}{r}$$

rearrange to isolate v_{esc}

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

plug in values

$$= \sqrt{\frac{(2)(6.672e^{-11})(5.683\times10^{26}\text{kg})}{5.8232\times10^{7}\text{m}}}$$

looked these up

$$= \boxed{3.609\times10^{4}\text{ m/s}}$$

2. Find $r_{orbital}$ for a satellite of Mars with a period T=9.00 hours. Mars: M= 6.39×10^{23} kg

First, lets draw the situation:

$9.00\,hr \cdot \frac{60\,min}{hr} \cdot \frac{60\,s}{min} = 3.24 \times 10^4\,s$

$3.3895 \times 10^6\,m$

looked up

circumference is d

$v = \frac{d}{t}$, $v = \frac{2\pi r}{T}$

How do we connect v and r

centripetal force is caused by gravitational force

$F_c = F_g$

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$v^2 = \frac{GM}{r}$

we want r to be only variable, so we replace v

$\left(\frac{2\pi r}{3.24 \times 10^4\,s}\right)^2 = \frac{GM}{r}$

looked up

$\frac{4\pi^2 r^3}{(3.24 \times 10^4)^2} = 6.672e^{-11} \cdot 6.39 \times 10^{23}\,kg$

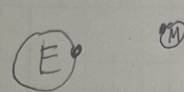
Solve for r

$r^3 = 1.13 \times 10^{21}$

$\boxed{r = 1.04 \times 10^7\,m}$

3. What is the change in energy of a 1000-kg payload taken from rest at the surface of Earth and placed at rest on the surface of the moon? What would be the answer if the payload were taken from the Moon's surface to Earth? Moon: M= 7.34×10^{22} kg r=1.737×10^6 m

First, let's draw the situation:



Now, I will compare the potential energy at each location:

Using universal law of gravitation

Plug in values

$U_E = \frac{-GM_e m}{r_e}$

$= \frac{(-6.672e^{-11})(5.97 \times 10^{24}\,kg)(1000\,kg)}{6.371 \times 10^6\,m}$

$U_E = -6.25e^{10}\,J$

$U_M = \frac{-GM_M m}{r_M}$

$U_M = \frac{(-6.672e^{-11})(7.34e^{22}\,kg)(1000\,kg)}{(1.737 \times 10^6\,m)}$

$U_M = -2.81e^9\,J$

Compare: $-6.25e^{10} - (-2.81e^9) = -6.00e^{10}\,J$

Answer: If the payload goes from Earth to the moon, the change in energy is $6.00e^{10}\,J$ and from the moon to Earth is $-6.00e^{10}\,J$.