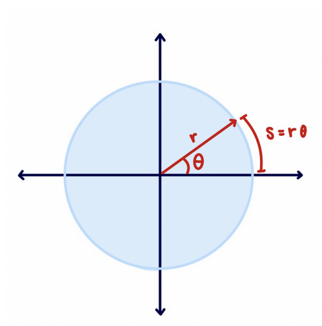
Lab: AP Review Sheets **AP Physics**
Chapter 10: Rotational Motion By: Kate Lim

Background / Summary
This review sheet will focus on Chapter 10, which covers angular/rotational kinetic energy, moments of intertia, and torque.

Major Topics
Angular/Rotational Kinematics
Angular kinematics functions the same as x-y kinematics, however, it uses a circular reference frame. The main difference is that angular kinematics is based on angular measurements, which uses radians as opposed to degrees.



Angular displacement (Δθ) - describes a particle's position by its radius *r* from the center of the circle at its angular position *θ* relative to the x-axis.	$\Delta\theta = \theta_f - \theta_i$
Angular velocity (ω) - refers to the change in angular position over a period of time and is calculated similarly to the x-y kinematics.	$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t}$ $\omega_{inst} = \frac{d\theta}{dt}$
Angular acceleration (α) - used to calculate the change in angular velocity over a period of time and is based on linear acceleration.	$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$ $\alpha_{inst} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

The relationship between linear & angular measurements

$$s = r\theta \;\&\; v = r\omega \;\&\; a = r\alpha$$

Direction of Rotation

RIGHT HAND RULE!! Taking your right hand, point your index finger in the positive x-direction, then bend your remaining fingers in the direction of the positive y-direction, leaving your thumb pointing in the z-direction (the third axis perpendicular to the other two planes).	RIGHT HAND RULE 2!! Take your right fingers and bend them in the direction of motion, leaving your thumb pointing in the direction of circular motion (aka z-axis).

Rotational Kinetic Energy: $K_{rotational} = \Sigma K_i \rightarrow K_{rot} = \Sigma \frac{1}{2} m_i v_i^2$

To find the total rotational kinetic energy, add up all the kinetic energies of the smaller masses of the object.

Moment of Inertia (for discrete masses)
Moment of inertia (*I*) refers to a mass's "resistance to changing its motion" (from crashwhite).

$$K_{rot} = \Sigma \tfrac{1}{2} m_i v_i^2$$
$$= \Sigma \tfrac{1}{2} m_i (r_i \omega)^2$$
$$= \tfrac{1}{2} (\Sigma \tfrac{1}{2} m_i r_i^2) \omega^2$$
↳ $\boxed{K_{rot} = \tfrac{1}{2} I\omega^2 \text{ where } I = \Sigma m_i r_i^2 \text{ aka "moment of inertia"}}$

Moments of Inertia (for continuous distributions of mass)

continuous distribution of mass

$$x_{cm} = \frac{1}{M} \Sigma m_i x_i = \frac{1}{M} \int x \, dm$$

★ $dm = \lambda dx$

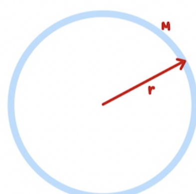
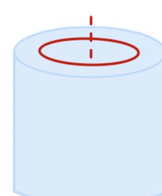
$dm = \sigma dA$

$dm = \rho dV$

moment of inertia for cont. distr. of mass

$$I = \Sigma m_i r_i^2$$
$$= \lim_{\Delta m_i \to 0} \Sigma \Delta m_i r_i^2$$
$$\boxed{I = \int r^2 \, dm}$$

Moments of Inertia for different situations

Hoop - thin, uniform hoop of mass M and radius R $I = MR^2$  $I = \int r^2 dm$ $= \int R^2 dm$ $= R^2 \int dm$ $= \boxed{MR^2}$	*Long thing rod about the center* - rod with a mass M & length L $I = \frac{1}{12} ML^2$  $I = \int r^2 dm$ & $dm = \lambda dr$ $= \int_{-L/2}^{L/2} r^2 \lambda dr$ $= \lambda \frac{1}{3} r^3 \Big\|_{-L/2}^{L/2}$ $= \frac{M}{L} \left(\frac{1}{3}(\frac{L}{2})^3 - \frac{1}{3}(\frac{-L}{2})^3\right)$ $= \boxed{\frac{1}{12} ML^2}$
Long thin rod about one end - rod with a mass M & length L $I = \frac{1}{3} ML^3$  $I = \int r^2 dm$ & $dm = \lambda dr$ $= \int_0^L r^2 \lambda dr$ $= \lambda \frac{1}{3} r^3 \Big\|_0^L$ $= \frac{M}{L} \left(\frac{L^3}{3}\right)$ $= \boxed{\frac{1}{3} ML^2}$	*Uniform cylinder* - uniform, solid cylinder with mass M, radius R, and length L $I = \frac{1}{2} MR^2$  $I = \int r^2 dm$ & $dm = \rho dV$ $= \int r^2 \rho dV$ & $dV = 2\pi r L dr$ $= \int r^2 \rho 2\pi r L dr$ $= 2\pi L\rho \int_0^R r^3 dr$ $= 2\pi L\rho \frac{1}{4} r^4 \Big\|_0^R$ $= \frac{1}{2}\pi L\rho R^4$ & $\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$ $= \frac{1}{2} L\pi R^4 \left(\frac{M}{\pi R^2 L}\right)$ $= \boxed{\frac{1}{2} MR^2}$

Parallel axis theorem: $I = I_{cm} + MD^2$

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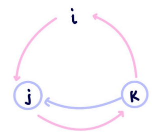
This theorem shows that if you know the moment of inertia about an object's center of mass, you can find the moment of inertia for any other parallel axis. The new axis of rotation must be parallel to I_{cm}.

<u>Torque</u>: $\tau = r \times F\sin\theta$ OR $\tau = Fd$ OR $\tau = r \times F$ *(UNITS = $N \cdot m$ or $\frac{kg \cdot m^2}{s}$)
The three main factors in determining the torque applied on an object is: the distance between the axis of rotation and the location of the F_{app} ($> r \Rightarrow >$"leverage"), the amount of F applied ($> F \Rightarrow > \tau$), and the angle between the radius and direction of F (as force is less perpendicular to the r, $\downarrow F_{app}$ & $F_{\parallel} = 0\tau$).

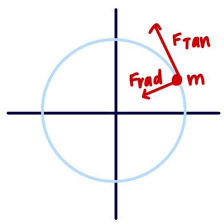
> **to find direction of torque, use the RHR with your index finger pointing in the direction of the radius, your other fingers bend in the direction of force, leaving your thumb pointing in the direction of torque
> (NOTE: +τ = CCW & -τ = CW)



ex. k x j = -i

*switch sign b/c opp. dir

<u>Cross-products with i, j vectors</u>
+i x +j = +k ⇔ -i x -j = -k

τ & α → if you apply a tangential force to the mass, you are applying a torque, which will cause the mass to have an angular acceleration $I = \Sigma m_i r_i^2 = mr^2$ $* \ \tau = r \times F = rF\sin\theta = rF_{tan}$ $\tau = rF_{tan}$ $= r(ma_{tan}) = r(m(\alpha r))$ $= mr^2\alpha = \boxed{I\alpha}$  	τ & $Work$ → a force at a certain radius from an axis of rotation causes a torque on the object, which does work on the object's rotation $dW = \vec{F} \cdot d\vec{s}$ $* \ s = r\phi \rightarrow ds = r\,d\phi$ ↳ $dW = F \cdot r\,d\phi$ $= F\sin\theta \cdot r\,d\phi$ $\tau = rF\sin\theta$ & $dW = \tau \cdot d\phi$ ↳ $W = \int \tau \cdot d\phi \rightarrow \boxed{W = \int_{x_i}^{x_f} F \cdot dx}$

Important Formulae

Degrees to radians: $1\,rev = 360° = 2\pi\,rads$	Arc length: $s = r\theta$	$dm = \lambda dx$ $dm = \sigma dA$ $dm = \rho dV$
Angular displacement (Δθ): $\Delta\theta = \theta_f - \theta_i$ $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	Average angular velocity (ω): $\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t}$	Average angular acceleration (α): $\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$
Angular velocity final (kinematics): $w_f^2 x = w_i^2 + 2\alpha\Delta\theta$	Instantaneous angular velocity (ω): $\omega_{inst} = \frac{d\theta}{dt}$	Instantaneous angular acceleration (α): $\alpha_{inst} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Rotational kinetic energy:		Moment of inertia:

<table>
<tr><td colspan="2">$K_{rot} = \Sigma \frac{1}{2} m_i v_i^2 \rightarrow \Sigma m_i (r_i \omega)^2 \rightarrow \frac{1}{2} I \omega^2$</td><td>$I = \Sigma m_i r_i^2$ OR $I = \int r^2 dm$</td></tr>
<tr><td>Parallel axis theorem:
$I = I_{cm} + MD^2$</td><td>Torque:
$\tau = r \times F sin\theta$ & $\tau = Fd$ & $\tau = r \times F$</td><td>Torque doing Work:
$W = \int \tau \cdot d\phi = \int_{x_i}^{x_f} F \cdot dx$</td></tr>
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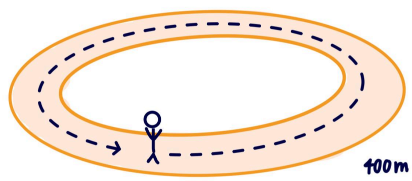
Problems:

1. A track star runs a 400-m race on a 400-m circular track in 45 s. What is his angular velocity assuming a constant speed? (#29 from textbook)



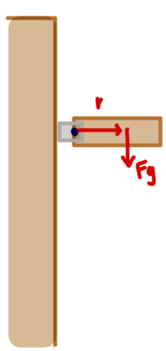
runs 400m race on 400m circular track in 45s.

$w = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{45 s} = \boxed{0.14 \text{ rad/s}}$

* use $\Delta\theta = 2\pi$ rad b/c running circular track

To find the angular velocity, use the equation $\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t}$, and then plug in the known values. We know that the runner takes 45 s to run a 400-m race on a 400-m circular track, so the Δt=45s and $\Delta\theta = 2\pi$ rads. We use 2π rads as the $\Delta\theta$ because he is running a circular track. Then plug values in the equation and solve.

2. A uniform rod of mass *M* and length *L* is attached to a wall by a pivot. What is its angular acceleration given the rod was released from rest (from its initial horizontal position). What is its linear acceleration on the right end of the rod? (problem from crashwhite)



$\tau = r \times F$

$\tau = I\alpha \rightarrow \alpha = \frac{\tau}{I}$

$\alpha = \frac{r \times F}{I} = \frac{rF_g}{I}$ & $I = \frac{1}{3}ML^2$

$\alpha = \frac{\frac{L}{2}(mg)}{\frac{1}{3}ML^2}$

$= \frac{\frac{L}{2}Mg}{\frac{1}{3}ML^2} = \boxed{\frac{3g}{2L}}$

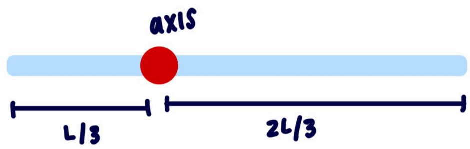
↳ $\alpha = \frac{3g}{2L}$ & $\alpha = ra$

$a = (L)\left(\frac{3g}{2L}\right)$

$= \boxed{\frac{3}{2}g}$

First solve for the angular acceleration using torque analysis. $\tau = r \times F$ & $\tau = I\alpha$, so when you combine the two equations together, you can find the angular acceleration. After you rearrange the equation, plug in the known values and solve. After you find the angular acceleration, you are bale to find the linear acceleration using the relationship between linear and angular acceleration: $\alpha = ra$. Plug in values and solve.

3. Calculate the moment of inertia by direct integration of a thin rod of mass M and length L about an axis through the rod at L/3, as shown below. Check your answer with the parallel-axis theorem. (#70 from textbook)



$I = \int r^2 \, dm$ & $dm = \lambda dr$

$= \int_0^L (r - L/3)^2 \lambda dr$

$= \int_0^L (r^2 - 2L/3\, r + L^2/9) \lambda dr$

$= \left(\frac{r^3}{3} - \frac{L}{3} r^2 + \frac{L^2}{9} r\right) \lambda \Big|_0^L$

$= \frac{M}{L}\left(\left(\frac{1}{3}(L)^3 - \frac{L}{3}(L)^2 + \frac{L^2}{9}(L)\right) - 0\right)$

$= \frac{M}{L}\left(\frac{L^3}{9}\right)$

$= \boxed{\frac{1}{9} ML^2}$

parallel axis theorem:

$I = I_{cm} + MD^2$

$I_{end} = \frac{1}{12} ML^2 + M\left(\frac{L}{6}\right)^2$

$= \frac{1}{2} ML^2 + \frac{1}{36} ML^2$

$= \frac{4}{36} ML^2$

$= \boxed{\frac{1}{9} ML^2}$

For this problem, first find the moment of inertia using the formula $I = \int r^2 dm$ and the relationship $dm = \lambda dx$. Take your integral from 0 to L, as that is the given length, and substitute r as r - L/3 because the rod is rotating around L/3. Then, finish the integration. To check your answer, use the parallel axis theorem $I = I_{cm} + MD^2$ with $I_{cm} = \frac{1}{12} ML^2$ which is for a rod rotated about the center. Then set the distance D as L/6 as that is the distance you need to move away from L/2 to get to L/3 (axis of rotation). Then if you get the same answer, you solved it correctly!