Lab: AP Review Sheets

Chapter 11: Angular Momentum

Jeffrey Liu

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1 Background

This chapter is about rolling objects, angular momentum, and conservation of angular momentum.

2 Vocabulary

- Rolling Objects: When we refer to "rolling objects," we refer specifically to objects which are *rolling without slipping*, meaning they remain in contact with the surface on which they are rolling at all times.
- Angular Momentum $\vec{L}$ ($\mathrm{kg\,m^2\,s^{-1}}$):
- Translational Motion: Movement resulting in displacement.
- Rotational Motion: Movement around an axis.

3 Topics

Rolling Objects

Rolling objects rotate as they move, with friction between the object and surface applying a torque that causes the object to rotate.

Equations modeling the motion of rolling objects can be found in Section 4.

The kinetic energy K of a rolling object is the sum of the object's *translational* and *rotational* kinetic energies.

$$K_{total} = K_{translational} + K_{rotational}$$
$$K_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

Angular Momentum

Angular momentum, like linear momentum, is conserved. Even objects moving in a straight line have an angular momentum relative to any point in space.

A particle of mass m, traveling with velocity v, has an angular momentum $\vec{L}$.

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = \vec{r} \times m\vec{v}$$
$$\vec{L} = rmv \sin\theta = I\omega$$

A particle moving in a straight line has an angular momentum of 0 relative to every point on its path. A particle moving perpendicular relative to the point of measurement will have an angular momentum rmv.

Angular momentum can also be related to torque.

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\frac{d}{dt}L = \frac{d}{dt}(\vec{r} \times \vec{p})$$
$$\frac{d}{dt}L = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

After substituting, we arrive at:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

For an extended body, such as a disk, we can apply $I_{disk} = \frac{1}{2}MR^2$

Conservation of Angular Momentum

The total angular momentum in a system is constant in both magnitude and direction if the net external torque on the system is zero. For a large system of particles:

$$\sum L_i = \sum L_f$$

For a system rotating about a fixed axis:

$$L_i = L_f$$
$$I_i\omega_i = I_f\omega_f$$

4 Formulae

- Tangential Displacement: $s = r\theta$
- Translational Distance: $x_{cm} = r\theta$
- Tangential Velocity: $v = r\omega$
- Translational Velocity: $v_{cm} = r\omega$
- Tangential Acceleration: $a = r\alpha$
- Translational Acceleration: $a_{cm} = r\alpha$
- Kinetic Energy: $K_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$
- Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$
- Moment of Inertia: $I = \frac{1}{2}mR^2$

5 Problems

5.1 Problem 1

A solid cylinder rolls up an incline at an angle of 20°. If it starts at the bottom with a speed of 10 m/s, how far up the incline does it travel?

Solution:

The kinetic energy of the cylinder at the bottom of the incline is equal to the sum of its translational kinetic energy and rotational kinetic energy:

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

We can substitute moment of inertia and angular velocity into this equation to find the initial kinetic energy:

$$K_i = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{1}{2}(\frac{1}{2}mR^2)(\frac{v}{R})$$

$$K_i = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

At the top of the incline, all initial kinetic energy is now gravitational potential energy.

Given that $P_i = 0$, we have:

$$P_f = \frac{3}{4}mv^2$$

Substituting $P_f = mgh$, we find:

$$h = \frac{3}{4}\frac{v^2}{g}$$

Given $v_i = 10m/s$, we can substitute to find:

$$h = \frac{3}{4}\frac{(10\,\mathrm{m/s})^2}{9.81\,\mathrm{m/s^2}}$$

$$h = 7.65\,\mathrm{m}$$

5.2 Problem 2

A 0.2 kg particle is traveling along the line y=2.0 m with a velocity 5.0 m/s. What is the angular momentum of the particle about the origin?

Solution:

We can use $\vec{L} = \vec{r} \times \vec{p}$ to determine the angular momentum of the particle.

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = 2.0\,\mathrm{m} \times (0.2\,\mathrm{kg})(5.0\,\mathrm{m/s})$$
$$\vec{L} = 5.0\,\mathrm{kg\,m\,s^{-1}}$$

5.3 Problem 3

A disk of mass 2.0 kg and radius 60 cm with a small mass of 0.05 kg attached at the edge is rotating at 2.0 rev/s. The small mass, while attached to the disk, slides gradually to the center of the disk. What is the disk's final rotation rate?

Solution:

In this problem, angular momentum is conserved:

$$L_i = L_f$$
$$I_i\omega_i = I_f\omega_f$$
$$\frac{1}{2}(2.0\,\mathrm{kg} + 0.05\,\mathrm{kg})(0.60\,\mathrm{m})^2(2.0\,\mathrm{rev/s}) = I_f\omega_f$$

When the small mass moves to the center of the disk, it no longer contributes to the moment of inertia, because it has no radius.

$$I_f = \frac{1}{2}(2.0\,\mathrm{kg})(0.60\,\mathrm{m})^2$$

We can substitute and solve for ω_f:

$$\frac{1}{2}(2.0\,\mathrm{kg} + 0.05\,\mathrm{kg})(0.60\,\mathrm{m})^2(2.0\,\mathrm{rev/s}) = \frac{1}{2}(2.0\,\mathrm{kg})(0.60\,\mathrm{m})^2\omega_f$$
$$\omega_f = 1.025\,\mathrm{rev/s}$$