

### Background | What to Expect

Oscillatory motion is fundamental to a wide range of natural phenomena and technological applications. From the vibrations of a violin string to the orbits of planets, understanding the principles of oscillation helps explain both the microscopic and macroscopic worlds. In this chapter, we will focus primarily on simple harmonic motion (SHM), a type of periodic motion that is particularly important for our AP physics exam.

#### Major Topics

1. Circular, Periodic Motion:
  - Understanding motion along a circular path as it relates to oscillatory motion.
  - Application of angular velocity and period in circular motion to analyze oscillatory systems.
2. Simple Harmonic Motion (SHM):
  - Exploration of systems that can be modeled by sinusoidal functions.
  - Mass-spring systems and pendulums as classical examples of SHM.
3. Oscillators:
  - Study of various physical systems that exhibit oscillatory behavior, including mechanical and electrical oscillators.

## Vocabulary & Formulae

### Vocabulary

- Amplitude ( $A$ ): The maximum extent of a vibration or oscillation, measured from the position of equilibrium.
- Period ( $T$ ): The time taken for one complete cycle of vibration to pass a given point.
- Angular Frequency ( $\omega$ ): The rate of change of the phase of a sinusoidal waveform, or the rate of change of the angle in a rotating system. Related to the frequency by  $\omega = 2\pi f$
- Phase Constant ( $\phi$ ): The initial angle of a sinusoidal function at time  $t=0$ .

### Important Formulas

Position in SHM:

- $x(t) = A \cos(\omega t + \phi)$

Velocity in SHM:

- $v(t) = -A\omega \sin(\omega t + \phi)$

Acceleration in SHM:

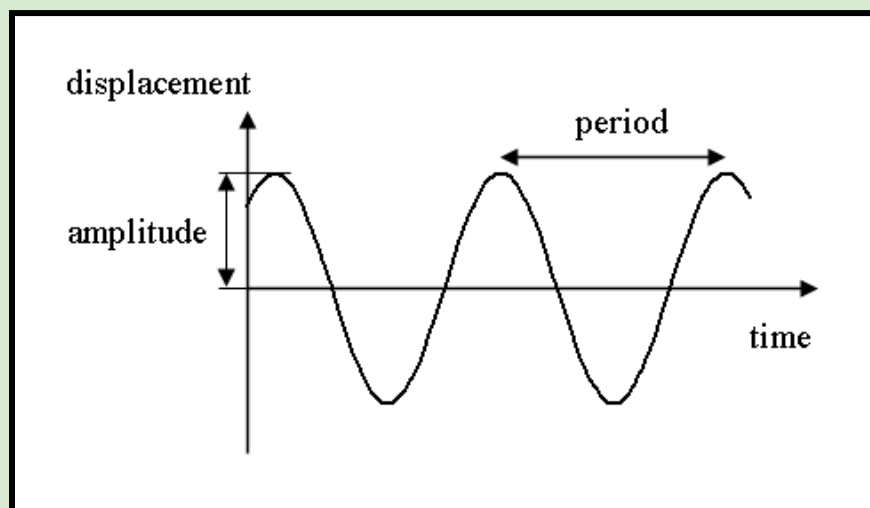
- $a(t) = -A\omega^2 \cos(\omega t + \phi)$

Period of a Mass-Spring System:

- $T = 2\pi m/k$

Period of a Simple Pendulum (Small Angle Approximation):

- $T = 2\pi L/g$



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### FRQs (3)

#### FRQ #1

A particle moves in a circular motion with a radius  $A$  of 0.5 m, starting from angular position  $\phi$ , and moves with a constant angular speed  $\omega$  of 2 rad/s.

(a) Calculate the  $x$ -coordinate of the particle at time  $t=1$  second using the cosine function.

**Solution:** The  $x$ -coordinate of a point moving in circular motion can be described by

$$x = A \cos(\omega t + \phi)$$

Assuming  $\phi=0$  for simplicity:

$$x = 0.5 \cos(2 \times 1 + 0) = 0.5 \cos(2) \approx 0.5 \times -0.416 = -0.208 \text{ m}$$

(b) Describe how the  $x$ -coordinate changes as the particle continues its motion.

**Solution:** As the particle continues in its circular motion, the  $x$ -coordinate oscillates back and forth between  $-A$  and  $A$  due to the cosine function. This oscillation repeats every  $2\pi/\omega$  seconds.

(c) Sketch the path of the particle over one complete cycle and mark the positions at  $t=0, \pi/2\omega, \pi/\omega$ , and  $3\pi/2\omega$

**Solution:** Draw a circle with center at the origin and radius  $A$ . Mark the starting point at  $t=0$  on the positive  $x$ -axis. As  $t$  increases, the particle moves counterclockwise, reaching the negative  $x$ -axis at  $\pi/\omega$ , completing the cycle back at the starting point at  $t=2\pi/\omega$ ,

**FRQ #2**

A mass of 0.5 kg is attached to a spring with a spring constant  $k=100\text{N/m}$ , set horizontally on a frictionless surface.

(a) If the mass is displaced 0.1 m from equilibrium and released, calculate the frequency of the motion.

**Solution:** The frequency  $f$  of a mass-spring system is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{100}{0.5}} = \frac{1}{2\pi} \sqrt{200} \approx 2.25 \text{ Hz}$$

(b) Determine the maximum speed of the mass.

**Solution:** The maximum speed  $v_{max}$  occurs at the equilibrium position and can be calculated using  $v_{max} = A\omega$  where

$$\omega = 2\pi f$$

$$v_{max} = 0.1 \times 2\pi \times 2.25 = 1.413 \text{ m/s}$$

(c) Sketch the energy distribution (potential and kinetic) of the system from the initial displacement to one full cycle.

**Solution:** Draw a graph with the total energy  $E$  (constant line) and curves for kinetic and potential energy. Kinetic energy peaks at equilibrium ( $x=0$ ) where potential energy is zero, and potential energy peaks at maximum displacement where kinetic energy is zero.

**FRQ #3**

A pendulum consists of a small bob of  $m=2\text{kg}$  attached to a string of length  $L=2\text{m}$ , swinging under the influence of gravity.

(a) Assuming small angle approximation, calculate the period of the pendulum.

**Solution:**

The period  $T$  of a simple pendulum is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$T = 2\pi \sqrt{\frac{2}{9.81}} \approx 2.837 \text{ s}$$

(b) Discuss how the period would change if the length of the string is doubled.

**Solution:**

Doubling the length of the string would increase the period since  $T$  is proportional to  $\sqrt{L}$ . Specifically,  $T' = 2\pi \sqrt{\frac{4}{9.81}} \approx 4.007 \text{ s}$

(c) Draw a free-body diagram of the bob at the lowest point in its swing.

**Solution:**

**Solution:** Draw the bob with two forces acting on it: gravitational force  $=mg$  directly downward and tension  $T$  in the upward direction along the line of the string (opposing the component of gravity).