

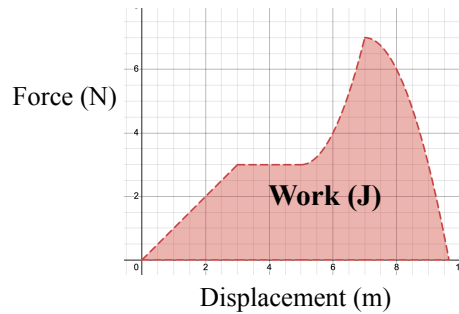
**Background/Summary**

This unit introduces work, kinetic energy, applied forces, springs, and power and how they are related by the work-energy theorem.

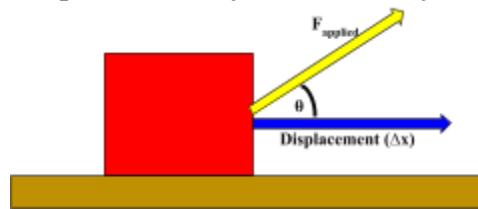
**Key Terms and Vocabulary**

1. Work is when energy is transferred to an object. This occurs when a force is applied over a displacement. Work is equivalent to change in energy.
2. The units for work are joules (J), which are equal to one Newton times one meter.
3.  $K$  is the kinetic energy of an object.
4.  $k$  is the spring-constant of a given spring, and represents the force per meter required to compress the spring.  $k$  is always positive, so in the spring force equation the force always opposes the direction it's being compressed.
5. The equilibrium position of a spring is when the system has zero net force.
6. Power is the rate at which work is done. To do work in less time requires more power.
7. The units for power are Watts, which are one Joule per second.

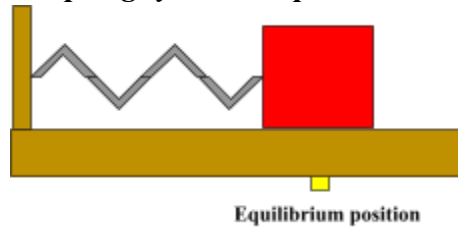
**Force vs. displacement graph**



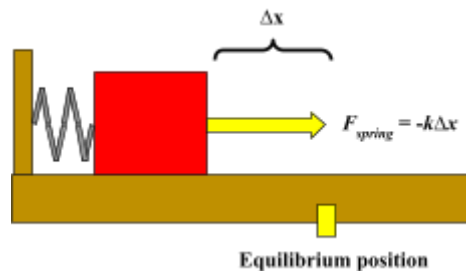
**Representation of work on an object**



**Spring system at equilibrium**



**Compressed spring system**



**Relevant Formulae**

$$W = F_x \Delta x$$

$$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$K = \frac{1}{2} m v^2$$

$$W = \Delta E$$

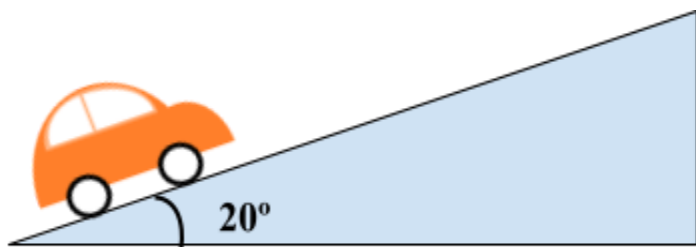
$$F_{spring} = -k\Delta x$$

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

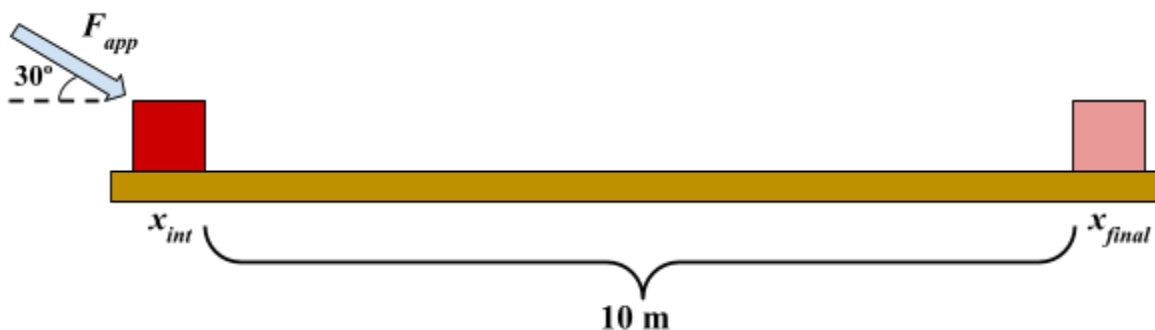
$$P_{instant} = \frac{dW}{dt}$$

$$P_{instant} = \vec{F} \cdot \vec{v}$$

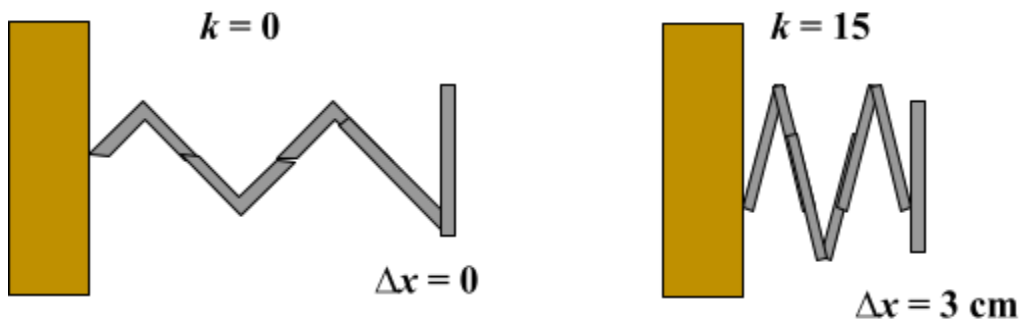
1. How much power does the engine of a 1200 kg car expend to drive up a  $20^\circ$  slope at 90 km/h?



2. A force is applied downwards at a  $30^\circ$  angle onto a 5 kg block resting on a table. The block accelerates constantly across the whole 10 meter table at  $3 \text{ m/s}^2$ . The  $\mu_{\text{kinetic}}$  between the block and table is 0.28. What is the work done by the applied force?

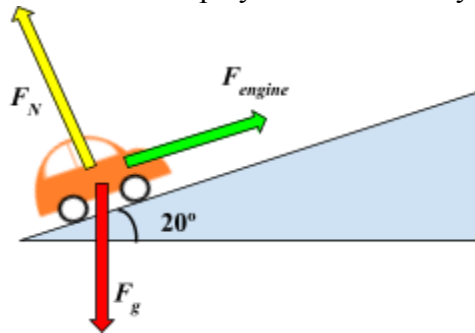


3. A spring has a spring-constant  $k = 500\Delta x$  where  $x = 0$  at the equilibrium position. Calculate the work that would need to be done to compress the spring 20 cm.

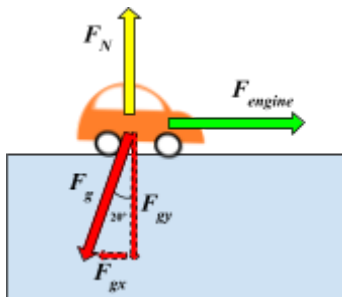


Solutions

1. Power can be calculated using  $P = \vec{F} \cdot \vec{v}$ . Because the car is not accelerating, the net force in the equation is zero. We can analyze the forces at play with a free body diagram.



This diagram can be rotated 20° to simplify the process of solving.



Now the vertical components are equivalent ( $F_N$  and  $F_{gy}$ ) and the vector of velocity is 90°. In the x direction,  $F_{engine}$  must equal  $F_{gx}$  or  $F_g \sin(20^\circ)$ .

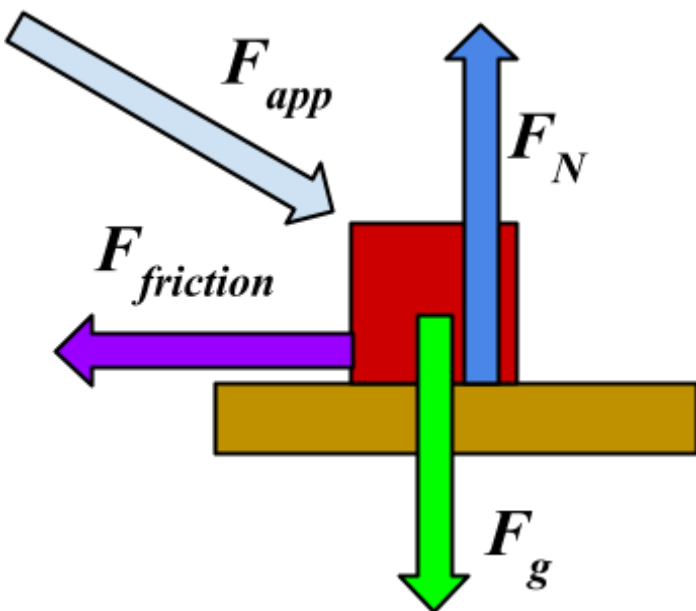
$$P = mg \sin(20^\circ) \cdot v \sin(90^\circ)$$

$$P = (1200)(9.8)\sin(20^\circ)(90 \times \frac{1000 \text{ m}}{\text{km}} \times \frac{\text{hr}}{3600 \text{ sec}})$$

$$P = 100554 \text{ W} \approx 101 \text{ kW}$$

2. With  $F = ma$ , we know that our net force is equal to mass times acceleration, which is 15 N.

Next, we draw a free body diagram to consider all of the forces at play.



It is easiest to start considering the forces in the y direction because they sum to zero.

$$F_N = F_g + F_{app-y}$$

$$F_N = mg + F_{app} \sin(30^\circ)$$

Then we consider forces in the x direction, where the net force is 15 N.

$$15 = F_{app-x} - F_{friction}$$

$$15 = F_{app} \cos(30^\circ) - \mu F_N$$

We can then plug the top equation into the bottom one to arrive at

$$15 = F_{app} \cos(30^\circ) - \mu(mg + F_{app} \sin(30^\circ))$$

Then solving for  $F_{app}$

$$15 + \mu mg = F_{app} \cos(30^\circ) - \mu F_{app} \sin(30^\circ)$$

$$F_{app} = \frac{15 + (0.28)(5)(9.8)}{\cos(30^\circ) - 0.28\sin(30^\circ)} = 39.6 \text{ N}$$

$$W = F\Delta x = 39.6(10) = 396 \text{ J}$$

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**Solutions**

3. The spring constant  $k$  in this problem is not consistent throughout the whole process of compressing the

spring. The two critical equations here are  $W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$  and  $F_{\text{spring}} = -k\Delta x$ .  $k = 500\Delta x$ , so  $F = -500\Delta x^2$ .

Plugging that into the integral, then integrating from  $x = 0$  to  $x = 0.2$ , we get:

$$W = - \int_0^{20} 500x^2 dx$$

$$W = - \left. \frac{500x^3}{3} \right|_0^{20}$$

$$W = -1300000 \text{ J}$$

This is the work done by the spring, which we can tell because the value is negative and the spring is the only force opposing the motion. As such, the force required to compress the spring 20 cm is the same but in the positive direction, so 1300000 J