Lab: AP Review Sheets
Chapter 15: Oscillation

AP Physics
by Roy Otamura

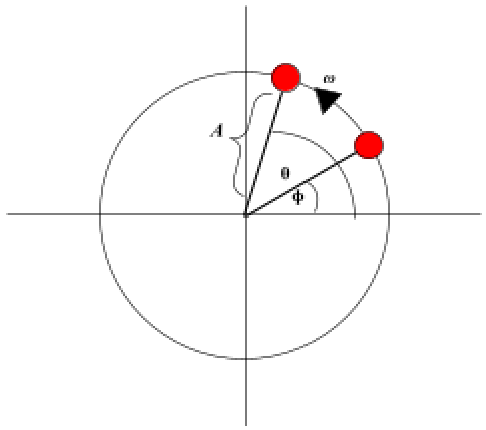
Background/Summary

This unit focuses on simple harmonic motion, exploring periodic motion and related equations.

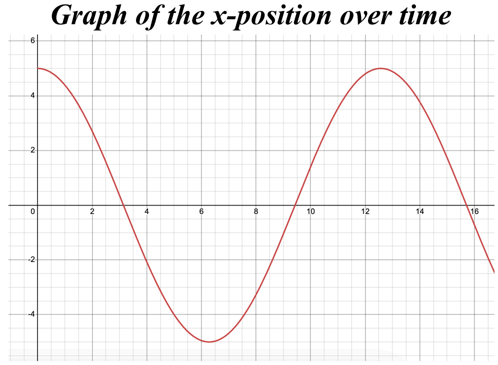
Key Terms and Vocabulary

1. Periodic motion for an object means that object moves in a repeating manner (like a pendulum swinging back and forth).
2. T represents the period for a system, or the amount of time it takes for one cycle.
3. f represents the frequency of a system, and is the inverse of the period. This means it represents the number of cycles in a second. The unit for frequency is Hz.
4. ω represents angular velocity.
5. A represents the amplitude of harmonic motion, or the maximum distance a system reaches away from zero.
6. ϕ represents the starting angular position of the system, or θ_i.
7. Position follows a cosine wave function in harmonic motion, like the x position as you go around a circle. The relationship between θ and ω is shown in the equations section.

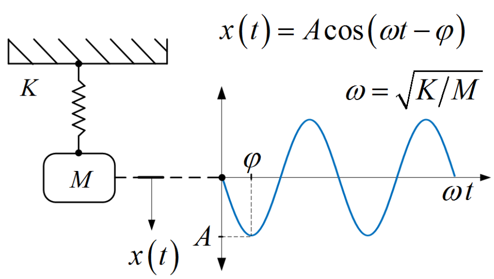
Representation of simple harmonic motion on the x-axis on a circle



Graph of the x-position over time



Comparison of motion to graph



Relevant Formulae

$T = \frac{2\pi}{\omega}$

$f = \frac{1}{T} = \frac{\omega}{2\pi}$

$\Delta\theta = \omega t$

$\theta_f = \omega t + \theta_i = \omega t + \phi$

$x = A\cos(\theta)$

$x = A\cos(\omega t + \phi)$

$v = \frac{d}{dt}A\cos(\omega t + \phi)$

$v = -\omega A\sin(\omega t + \phi)$

$v_{max} = -\omega A$

$a = -\frac{d}{dt}\omega A\sin(\omega t + \phi)$

$a = -\omega^2 A\cos(\omega t + \phi)$

$a = -\omega^2 x$

$a_{max} = -\omega^2 A$

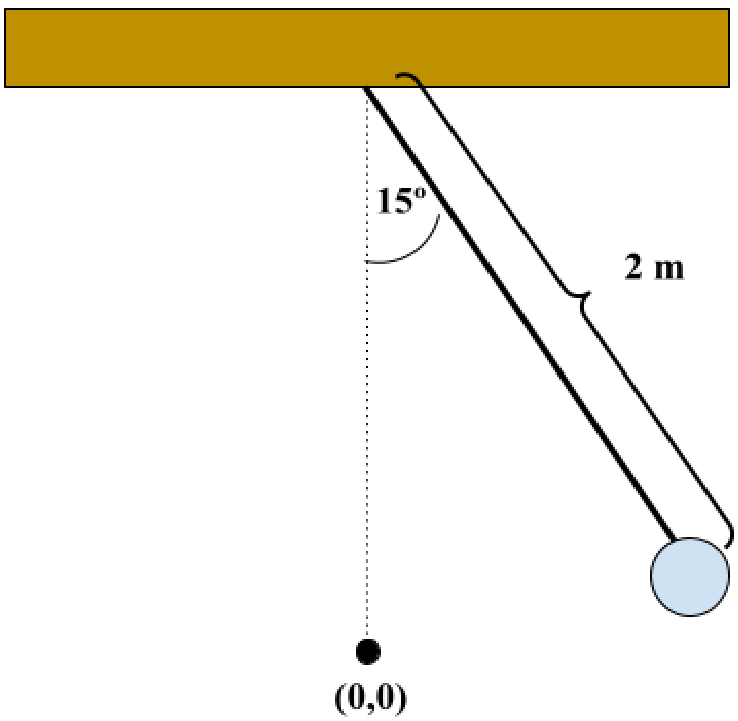
$\omega_{spring} = \sqrt{\frac{k}{m}}$

$\omega_{pendulum} = \sqrt{\frac{g}{L}}$

$\omega_{physical\ pendulum} = \sqrt{\frac{rmg}{I}}$

1. A block of mass *m* is attached to a spring with a spring constant *k*, and when the spring is compressed and then released, the system demonstrates simple harmonic motion.
   a. Prove that $\omega = \sqrt{\frac{k}{m}}$.
   b. When the spring is released, it reaches a maximum velocity of 5 m/s. How far was the spring initially compressed? Answer is terms of *k, m,* and other physical constants.

2. A sphere of mass *m* is attached to the ceiling by a weightless rope of length *L*. When the mass is pulled back and then allowed to swing, the system demonstrates simple harmonic motion
   a. Prove that $\omega = \sqrt{\frac{g}{L}}$
   b. Say the length of the rope is 2 meters and the pendulum is released from the point indicated in the diagram below. After 5 seconds, where is the mass? Your answer should consist of an *x* and *y* coordinate where the bottom of the swing is (0,0)



3. A rod of mass *m* and length *L* hangs from the ceiling by a pivot. When the rod is lifted and then released, it swings back and forth, demonstrating simple harmonic motion.
   a. Prove that $\omega = \sqrt{\frac{Lmg}{2I}}$
   b. Let *m* be 2.0 kg and *L* be 3 meters. Calculate ω in terms of *m, L,* and physical constants.

Solutions

1.

a. The only relevant force in this equation is the force of the spring, because gravity and friction are being ignored. As such $F_{\text{net}} = F_{spring}$ and $ma = -kx$

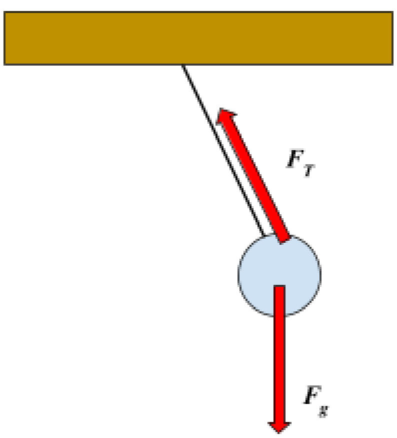
$$a = -\frac{k}{m}x$$
$$a = -\omega^2 x$$
$$\omega^2 x = \frac{k}{m}x$$
$$\omega = \sqrt{\frac{k}{m}}$$

b. The maximum velocity is 5 m/s. Considering $v_{max} = -\omega A$, we can solve for A, or the maximum distance the spring is compressed.

$$\frac{v}{\omega} = -A$$
$$A = 5\sqrt{\frac{m}{k}}$$

2.

a. Begin with $F_{\text{net}} = ma$, then consider the forces at play in a free body diagram.

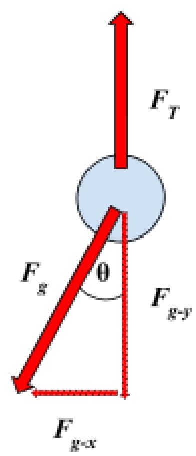


We can simplify the process by tilting our axes so that the force of tension points up and then the force of gravity will be split into x and y components.

The rotated diagram shows that $\Sigma F_x = F_{gx} = F_g \sin(\theta)$.

Setting this equal to $F_{\text{net}} = ma$, we get:

$$ma = -mg\sin(\theta)$$
$$a = -g\sin(\theta)$$
$$a = \frac{d^2 s}{dt^2} = \frac{d^2 L\theta}{dt^2}$$
$$\frac{-g}{L}\sin(\theta) = \frac{d^2\theta}{dt^2}$$



At small angles, $\sin(\theta) \approx \theta$ so for simple harmonic motion we simply substitute in θ

$$\frac{-g}{L}\theta = \frac{d^2\theta}{dt^2}$$
$$\frac{-g}{L} = \omega^2$$
$$\omega = \sqrt{\frac{g}{L}}$$

b. L = 2 m, so $\omega = \sqrt{\frac{g}{2}} = 2.21$.

$f = \frac{\omega}{2\pi} = 0.35$ cycles/second

5 seconds $\cdot \frac{0.35\ cycles}{second} = 1.75$ cycles

After 1 cycle the mass is at the same position, then after 0.75 cycles, the mass goes all the way to the other side and then comes back to the bottom, the equivalent of ¾ of a full cycle. So, the mass is at (0,0)

3.

a. Instead of using $F = ma$, this time we use $\tau = r \times F = I\alpha$. The force at play is gravity, which is applied at $\sin(\theta)$.

$I\alpha = rmg \sin(\theta)$

The center of gravity is at $\frac{L}{2}$, so we use that for r.

$\alpha = \frac{Lmg}{2I} \sin(\theta)$

Again we use $\sin(\theta) \approx \theta$.

$\alpha = \frac{Lmg}{2I}\theta$

$\omega^2 = \frac{Lmg}{2I}$

$\omega = \sqrt{\frac{Lmg}{2I}}$

b. m is now 2.0 kg and L is 3.0 meters.

$I = \frac{1}{3}mL^2$

$\omega = \sqrt{\frac{3g}{2L}}$

$\omega = \sqrt{\frac{g}{2}}$