

**Background/Summary:**

Oscillation deals with studying repetitive, back-and-forth motion in various physical systems. It is a fundamental concept in physics and applies to many natural phenomena and engineering applications. Oscillation is based on Hooke's Law ( $F = -kx$ ).

**General Equations:** All these equations will be used for all the topics in this unit. However, the only variation is the angular velocity.

- $x = A\cos(\omega t + \phi)$
- $v = -\omega A\sin(\omega t + \phi)$ , where  $v_{\max} = -\omega A$
- $a = -\omega^2 A\cos(\omega t + \phi)$ , also  $a = -\omega^2 x$
- Period( $T$ ) =  $2\pi/\omega$
- Frequency( $F$ ) =  $1/T = \omega/2\pi$

**Major Topics:**

Circular, Periodic Motion: Motion where an object moves around a fixed point with a constant radius.

Simple Harmonic Motion: Where an object oscillates back and forth around an equilibrium position. The magnitude of its acceleration is directly proportional to its displacement from the equilibrium position and is always directed towards it.

- $\omega = (k/m)^{1/2}$
- Angular velocity =  $(\text{spring constant}/\text{mass})^{1/2}$

Mass-Spring System: This type of oscillation involves a spring and mass constantly fighting to reach equilibrium. The system's behavior depends on the spring constant and the mass attached to it. The difference from SHM is the angular velocity plugged into previous equations.

- $\omega = (k/m)^{1/2}$
- Angular velocity =  $(\text{spring constant}/\text{mass})^{1/2}$

Simple Pendulum: Consists of a mass (called the bob) attached to a string or rod of negligible mass, which is fixed at a pivot point and follows simple harmonic motion.

- $\omega = (g/L)^{1/2}$
- Angular velocity =  $(\text{acceleration due to gravity}/\text{length})^{1/2}$

Physical Pendulum: This is similar to a simple pendulum but consists of a rigid body with a mass attached. This body is free to rotate around a fixed axis of rotation.

- Oscillation:  $\omega = (rmg/I)^{1/2}$
- Angular velocity =  $((\text{radius} \times \text{mass} \times \text{gravitational acceleration})/\text{moment of inertia})^{1/2}$

(Easy) A mass-spring system has a spring constant of 20 N/m. If a 0.5 kg mass is attached to the spring and stretched 0.2 meters from its equilibrium position, calculate the force exerted by the spring and the system's frequency of oscillation.

**Solution:**

Stating the known values:

$$k = 20 \text{ N/m}$$

$$m = 0.5 \text{ kg}$$

$$x = 0.2 \text{ m}$$

Use Hooke's Law to calculate the force exerted by the spring constant and displacement:

$$F = -kx$$

$$F = -(20)(0.2)$$

$$F = -4 \text{ N}$$

Now, calculate the frequency of oscillation of the system by plugging in the known values:

$$f = 1/T = (k/m)^{1/2} / 2\pi$$

$$f = (1/2\pi)(20/0.5)^{1/2}$$

$$f = \underline{1.01 \text{ Hz}}$$

(Medium) It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective force constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?

**Solution:**

Part (A): Rearrange Hooke's Law to solve/isolate for the spring constant:

$$F = -kx$$

$$k = F/x$$

The force in the problem is equal to  $mg$ , setting you up to plug in the known values and solve for the spring constant:

$$k = -mg/-x$$

$$k = (120)(9.8)/(0.75 \times 10^{-2})$$

$$k = \underline{1.57 \times 10^5 \text{ N/m}}$$

Part (B): The question asks to find the mass if the spring is compressed 0.48 cm, which allows us to use the same formula as above:

$$k = -mg/-x$$

But now, we are solving for mass, so we need to isolate it, and then we will be able to plug in our known values:

$$k = -mg/-x$$

$$m = kx/g$$

$$m = (1.57 \times 10^5)(0.48 \times 10^{-2})/(9.8)$$

$$m = \underline{76.9 \text{ kg}}$$

Since his mass is 76.9 kg, the person can play on the under-85 kg team.

(Hard) The device entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring. (a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its force constant? (b) What is the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m? Hint: draw a visual of the problem.



**Solution:**

Part (A): Rearrange Hooke's Law to solve/isolate for the spring constant:

$$F = -kx$$

$$k = F/x$$

For this problem, the force is equal to the weight of the child (mg):

$$k = -mg/-x$$

$$k = -8(9.8)/-(0.250)$$

$$k = \underline{314 \text{ N/m}}$$

Part (B): Using the period equation, we can calculate the time for one bounce:

$$T = 2\pi(m/k)^{1/2}$$

$$T = 2\pi(8/314)^{1/2}$$

$$T = \underline{1.00 \text{ sec}}$$

Part (C): From the equations provided, we can combine the equations to solve for  $v_{\max}$ :

$$\omega = (k/m)^{1/2}$$

$$v_{\max} = -\omega A$$

$$v_{\max} = A(k/m)^{1/2}$$

With the equation derived above, we can plug in the known values to solve for  $v_{\max}$ :

$$v_{\max} = 0.2(314/8)^{1/2}$$

$$v_{\max} = \underline{1.25 \text{ m/s}}$$