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Chapter 6: Circular Motion

Background and Major Topics

The focus of this unit is the centripetal force, which makes objects move in circular paths. Other topics include friction forces and drag forces, which oppose the relative motion of objects. Compared to previous problems, centripetal force problems tend to require more complex free-body diagrams, and friction force problems tend to address more realistic situations.

Vocabulary

The **friction force** is a force that opposes the relative motion of an object. The key type of friction in this course is **sliding friction**, a force that exists between two surfaces in contact with each other. Sliding friction depends on the **coefficient of friction μ**, a measure of the "stickiness" of the two surfaces, and the normal force, which determines how much the surfaces are being "compressed." There is a **static coefficient of friction μ_s** when the surfaces are not moving relative to each other (slipping) and a **kinetic coefficient of friction μ_k** when they are. The static coefficient of friction can only calculate the *maximum* static friction. The maximum static friction is greater than the maximum kinetic friction because upon moving the surfaces "unlock."

The **centripetal force** is a force that causes objects to move in circular paths. Specifically, it causes objects to experience a "center-seeking", **radial**, or **centripetal acceleration**. Many different forces can be a centripetal force, including friction. The motion of objects that experience both radial and tangential acceleration can be denoted using **hat-theta and hat-r** unit vector notation; the net acceleration is the vector sum of the two. There is no "centrifugal" force, only inertia.

The **drag** or **resistive force** is a force that varies either linearly or exponentially according to an object's velocity through a fluid. **Terminal velocity** is the velocity reached by a free-falling object when the forces of gravity and air resistance are equal.

Formulae

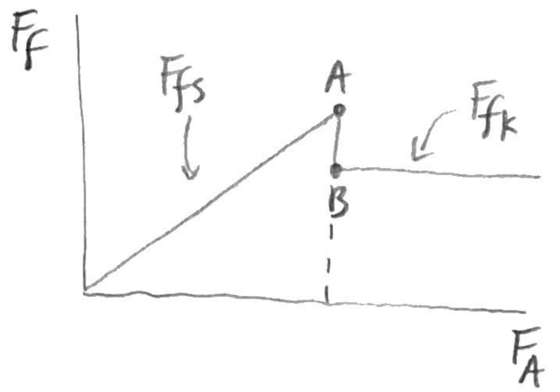
$F_f = \mu F_N$ (General) $\quad F_{fs} \le \mu_s F_N$ (Static) $\quad F_{fk} = \mu_k F_N$ (Kinetic)

$\Sigma F_c = \frac{mv^2}{r} \quad a_c = \frac{v^2}{r}$

$R = -bv$ (Linear) $\quad R = \frac{1}{2} D\rho A v^2$ (Quadratic)

Diagrams

1. The relationship b/w static and kinetic friction

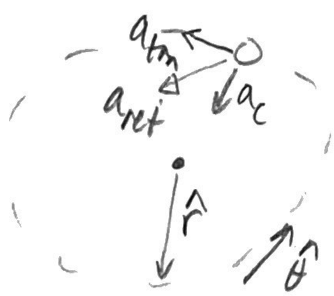


$A = \mu_s F_N$

$B = \mu_k F_N$

Note that $A > B$.

2. Non-uniform circular motion and the unit vectors

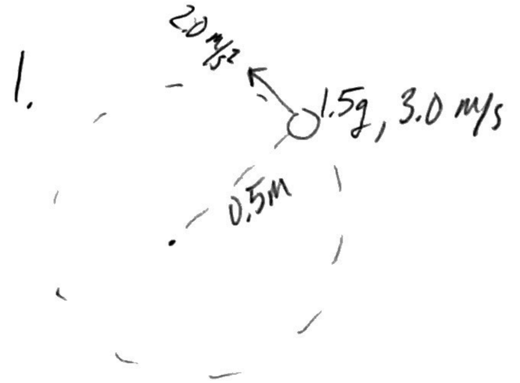


Problems

1. (Easy) A 1.5 g particle moves in a circle with a velocity of 3.0 m/s and a tangential acceleration of 2.0 m/s^2 about a point 0.5 m away from it. What is the net force on the particle? Express your answer in unit vector notation.
2. (Medium) A cylindrical exploding bullet with cross-sectional area A, mass m, and initial velocity v_i travels through fresh-squeezed orange juice with drag coefficient D and density p. The resistive force of the juice on the bullet varies according to the square of the bullet's velocity. Write and solve a differential equation in order to find the bullet's velocity at a time t.
3. (Hard) A rocket-powered 1000 kg racecar rolls without slipping through a track consisting of a horizontal loop and a vertical loop at a constant velocity of 120 km/hr. Both loops have a radius of 150 m.
    a. What is the coefficient of friction between the tires of the racecar and the track of the horizontal loop? Is it a coefficient of static or kinetic friction? Justify your answer.
    b. When, after starting from the very bottom of the vertical loop, the racecar has traveled a third of distance around the loop, how much force must the track exert on the racecar to keep it moving on the track?

Solutions

1.



$m = 1.5 \cdot 10^{-3}$ kg

$v = 3.0$ m/s

$a_{tan} = 2.0$ m/s²

$r = 0.5$ m

$$\Sigma F = F_{tan} + F_{rad}$$
$$= m(a_{tan} + a_{rad})$$

We know m and a_{tan}, so we just need a_{rad}. $a_c = \frac{v^2}{r} = \frac{(3)^2}{(0.5)} = 18$ m/s²

Substituting, $\Sigma F = (1.5 \cdot 10^{-3})(2.0\hat{\theta} - 18\hat{r}) = \boxed{(3.0 \cdot 10^{-3}\hat{\theta} - 27 \cdot 10^{-3}\hat{r})\,N}$

Note that $\hat{r}$ is negative for a_c b/c $\hat{r}$ points away from the center of the path, while a_c points towards it.

2. The resistive force is the only force, so we'll use $R = \frac{1}{2}D\rho A v^2$ and $\Sigma F = ma$, substituting $\frac{dv}{dt}$ for a.

$$\frac{1}{2}D\rho A v^2 = m\frac{dv}{dt} \rightarrow \int_{v_i}^{v_f} \frac{1}{v^2}\,dv = \int_0^t \frac{D\rho A}{2m}\,dt \rightarrow -\frac{1}{v}\Big|_{v_i}^{v_f} = \frac{D\rho A t}{2m} \rightarrow$$

$$-\frac{1}{v_f} = \frac{D\rho A t}{2m} - \frac{1}{v_i} \rightarrow \boxed{v_f = \frac{2mv_i}{2m - D\rho A t v_i}}$$

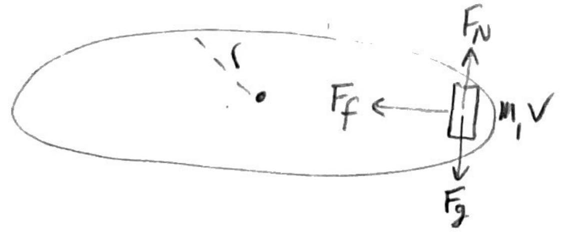
3.

$m = 1000$ kg

$v = 120 \frac{km}{hr} \cdot \frac{hr}{3600 s} \cdot \frac{10^3 m}{km} = 33.3$ m/s

$r = 150$ m

a.



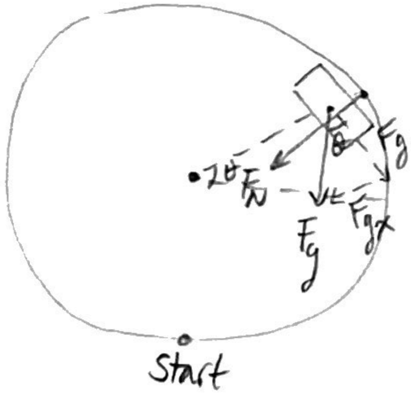
$\Sigma F_y = 0$, so $F_g = F_N$.

Friction supplies the centripetal force, so $\Sigma F_c = F_f$.

$$\Sigma F_c = F_f \rightarrow \frac{mv^2}{r} = \mu F_N \rightarrow \frac{\cancel{m}v^2}{r} = \mu \cancel{m}g \rightarrow \mu = \frac{v^2}{rg} = \frac{(33.3)^2}{(150)(9.8)} = \boxed{0.754}$$

This is a coefficient of $\boxed{\text{static friction.}}$ The wheels of the racecar roll "without slipping," so there is no relative movement b/w the contact point of the tire and the road.

b.



The racecar is at 30° above the horizontal $\left(-90° + \frac{360°}{3} = 30°\right)$.

At this point, both the horizontal component of the force of gravity and the normal force of the track on the racecar supply a centripetal force.

$$\Sigma F_c = F_{gx} + F_N \rightarrow \frac{mv^2}{r} = mg\sin\theta + F_N \rightarrow F_N = \frac{mv^2}{r} - mg\sin\theta$$

$$= \frac{(1000)(120)^2}{(150)} - (1000)(9.8)\sin(30°) = \boxed{9.11 \cdot 10^4\ N}$$