Lab: AP Review Sheets **AP Physics**

Chapter 8: Potential Energy & Conservation of Energy **By Carolyn Wu**

Background / Summary

In this unit, we learned how to apply kinetic energy, potential energy, and conservative and non-conservative forces to solve problems of moving objects.

Major Topics:

Potential energy: aka stored energy that has the ability to be converted or used later on

Gravitational potential energy: the energy that the gravitational field between two masses provides; often is turned into kinetic energy

Elastic potential energy: the energy that is stored in a spring system when the spring is moved outside of its equilibrium; also often turned into kinetic energy

TLDR on work vs potential energy:

$$W \text{ done by gravity} = U_i - U_f$$
$$W_g = -\Delta U$$
$$W_g = \int_{x_i}^{x_f} F_g \cdot dx$$
$$\Delta U = -\int_{x_i}^{x_f} F_g \cdot dx$$

$$W \text{ done by spring} = U_i - U_f$$
$$W_s = -\Delta U$$
$$W_s = \int_{x_i}^{x_f} F_{spring} \cdot dx$$
$$\Delta U = -\int_{x_i}^{x_f} F_s \cdot ds$$

Basically, any type of work that is being done on an object (gravitational or elastic) is taking away from its potential energy. Thus, whatever loss of potential energy in the object has to equal the value of the work. (the magnitudes are the same)

Derive gravitational potential energy:

$$W_g = \int_{y_i}^{y_f} F_g \cdot dy = \int_{y_i}^{y_f} -mg \cdot dy = -mg \int_{y_i}^{y_f} dy = -mg\,(y_f - y_i)$$
$$W_g = mgy_i - mgy_f$$
$$W_g = mgh_i - mgh_f \rightarrow \star\ U_g = mgh$$
$$W_g = U_i - U_f = -\Delta U$$

Derive elastic potential energy:

$$W_s = \int F_s \cdot dx = \int_{x_i}^{x_f} -kx \cdot dx = -k\int_{x_i}^{x_f} x\,dx$$
$$(\text{bc } F_s = -kx)$$
$$W = -\frac{1}{2}kx^2\Big|_{x_i}^{x_f} = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 \rightarrow \{\ U = \frac{1}{2}kx^2$$
$$\text{so } W_s = U_i - U_f = -\Delta U_s$$

Key points:

- For gravitational potential: You can define your starting height (h=0) anywhere; the other heights you use in calculations will be relative to that starting point.
- For elastic potential: measure your displacement as the distance from the unstretched equilibrium position
- If gravity or spring is doing **work** then it is **lowering potential energy**; this means equations are structured as shown above in work vs potential energy

Conservative Forces = The work doesn't rely on the path of the particle, only the displacement

- So work done by gravity is a conservative force

Nonconservative Forces = if the force impacts the mechanical energy, basically if you moved an object somewhere, and then back to its original position, net work ≠ 0,

- i.e. frictional forces

Energy and kinetic friction:
If the force of kinetic friction acts on a moving object, then it induces negative work:

$$W = F \cdot x$$
$$= Fx \cos\theta = F_{friction}(x)\cos(180) = -F_{friction}\, x$$

We can calculate energy changes using: $\Delta E_{internal} = f_k d$

Conservation of energy:
Total mechanical energy = kinetic + gravitational potential + elastic potential energy
Total mech E will stay constant if the objects are in an isolated system (no outside forces)

Conservation of *Mech E:* If system is isolated, then you can set added initial energies equal to added final energies:

$$\sum E_{initial} = \sum E_{final}$$
$$K_i + U_i = K_f + U_f$$
$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

Conservation of *Total E*: Even in a non-isolated system, if you account for outside force doing work, and any change in internal energy, then you can still set up an equation:

$$\sum E_{initial} = \sum E_{final}$$
$$\sum W_{ext} + K_i + U_i = K_f + U_f + \Delta E_{int}$$

Quick note on conservative force and potential energy:
We know how to integrate to find work & that work = negative potential energy. We can put that together and express it as a differential:

$$W \text{ done by a force} = \int_{x_i}^{x_f} F \cdot dx$$
$$W \text{ by conservative force} = \int_{x_i}^{x_f} F_{cons} \cdot dx$$
$$\int_{x_i}^{x_f} F_{cons} \cdot dx = U_i - U_f = -\Delta U$$
$$F_{cons} \cdot dx = -dU$$
$$F = -\frac{dU}{dx}$$

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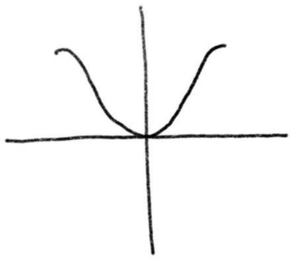
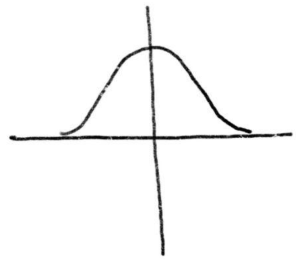
Note on stable vs unstable equilibrium:

- When looking at a graph of potential energy as a function of displacement (y-axis=potential energy U; x-axis=displacement X), points of equilibrium are when the slope of the function = 0
- If the curve is concave up, that displays stable equilibrium; concave down displays unstable equilibrium



stable equilibrium



unstable equilibrium

Necessary equations+when to use:

Use when in an isolated system

$$K_i + U_i = K_f + U_f$$

Use when in an nonisolated system (friction, external force etc)

$$W_{ext} + K_i + U_i = K_f + U_f + \Delta E_{int}$$

$$\Delta E_{int} = f_k d$$

$$W_{ext} = F_{app} X$$

Needed when calculating conservation of total energy

When given forces over an interval or potential energies and you need to find work

$$W = \int_{x_1}^{x_f} F \cdot dx$$

$$W = -\Delta U$$

When you need to find conservative force and are given info about potential energy with respect to position

$$F_{conservative\ force} = \frac{-dU}{dx}$$

Problems:

1. [easy] A force f(x)=(-5x2+7x)N acts on a particle. How much work does the force do on the particle as it moves from x=2.0m to x=5.0m?
2. [medium] Assume that the force of a bow on an arrow behaves like the spring force. In aiming the arrow, an archer pulls the bow back 50 cm and holds it in position with a force of 150N. If the mass of the arrow is 50g and the "spring" is massless, what is the speed of the arrow immediately after it leaves the bow?
3. [hard] A baseball of mass 0.25 kg is hit at home plate with a speed of 40 m/s. When it lands in a seat in the left-field bleachers a horizontal distance 120 m from home plate, it is moving at 30 m/s. If the ball lands 20 m above the spot where it was hit, how much work is done on it by air resistance?

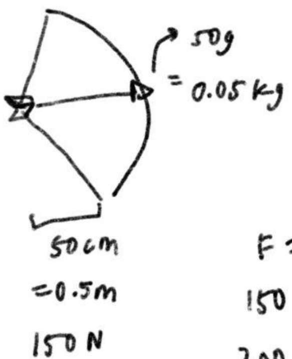
Solutions + Blurbs:

1) $F(x) = (-5.0x^2 + 7.0x)$ N

$$W = \int F\,dx = \int_2^5 (-5.0x^2 + 7.0x)\,dx \rightarrow \text{plug into calc} = \boxed{-121.5\ J}$$

Explanation: To find work done by a force, you just take the integral of that force and plug in the bounds specified by the question, in this case [2,5]

2)



$$U_{si} + K_i = U_{sf} + K_f$$

$$\frac{1}{2}kx^2 + \cancel{\frac{1}{2}mv^2} = \cancel{\frac{1}{2}kx^2} + \frac{1}{2}mv^2$$

$F = kx$

$150 = k(0.5)$

$300 = k$

$$\cancel{\frac{1}{2}}(300)(0.5)^2 = \cancel{\frac{1}{2}}(0.05)(v)^2$$

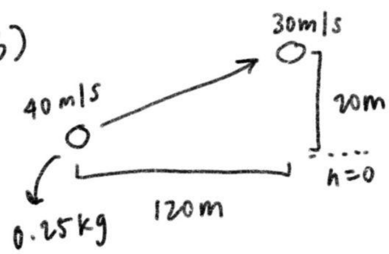
$$(300)(0.25) = (0.05)(v^2)$$

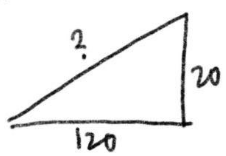
$$v^2 = 1500$$

$$v = \sqrt{1500} = \boxed{38.73\ m/s}$$

Explanation: We start with the basic equation of the conservation of mechanical energy since it's an isolated system. Instead of gravitational potential energy, make sure to use elastic potential energy. From Hooke's law, we know that F=kx, and we know the F to keep the arrow in position is 150. So plug 150N and 0.5m into F=kx to get the spring constant. Once we have the spring constant, we have all the variables we need for the conservation of the mechanical energy formula. Plug in all values, and isolate velocity to find the speed of the arrow immediately after it leaves the bow. We can ignore initial kinetic energy because the arrow is not moving, and we can ignore final elastic potential since the spring/bow has returned to equilibrium.

3)





$$\sqrt{120^2 + 20^2} = ? = 121.66m$$

$$\cancel{U_{gi}} + K_i = U_{gf} + K_f + \Delta E_{int}$$

$$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mv^2 + f_k d$$

$$\frac{1}{2}(0.25)(40)^2 = (0.25)(9.8)(20) + \frac{1}{2}(0.25)(30)^2 + f_k(121.66m)$$

$$200 = 49 + 112.5 + f_k(121.66)$$

$$200 = 161.5 + \underbrace{f_k(121.66)}$$

$$f_k = 0.3165$$

$$W = f_k d \rightarrow (0.3165)(121.66) = \boxed{38.5\ J}$$

Explanation: The problem wants us to find the work done by air resistance, so we already know that we start with the conservation of total energy equation, not the conservation of mechanical energy. We should find the displacement the ball traveled, which we will need later on. You find that with the Pythagorean theorem, and you use the x and y distance given. After setting up the conservation of energy equation, plug in all the values we know, which leaves us with the force of air resistance. We know that work=fd, so with the force, we can multiply by the distance it traveled, which in this case is 121.66, leaving us with 38.5 joules.

Technically, you don't even need to find the individual force of air resistance, since we just want work. So you can also just do 200-161.5 to isolate change of internal energy, resulting the same answer.