Lab: AP Review Sheets **AP Physics**

Chapter 10: Rotation of a Rigid Object About a Fixed Axis **By Carolyn Wu**

Background / Summary

In this unit, we learned how to use rotational kinematics, derive moments of inertia, and use Torque to solve problems.

Major Topics:

Establishing basics:

Angular velocity = "omega" ω

Angular acceleration = "alpha" α

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t} \qquad \alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$$

$$\omega_{inst} = \frac{d\theta}{dt} \qquad \alpha_{inst} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Relationship between linear and angular measurements: s=rθ; v=rω; a=rα

rotational kinematics formulae:

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \qquad \alpha_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Right hand rule review: to figure out the direction of circular motion, bend your four fingers in the direction of rotation, then your thumb is pointing in the direction of circular motion.

Rotational kinetic energy: the basic concept kinetic energy is that were adding up multiple kinetic energies of multiple smaller masses inside a bigger mass to get the overall kinetic energy. But instead of regular velocity, we have to use "omega"

$$K_{rotational} = \sum K_i \qquad K_{rot} = \sum \frac{1}{2} m_i v_i^2$$

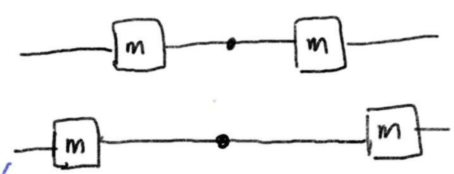
$$K_{rot} = \sum \frac{1}{2} m_i v_i^2 \qquad K_{rot} = \sum \frac{1}{2} m_i (r_i \omega)^2 \qquad K_{rot} = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$$K_{rot} = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2 \qquad K_{rot} = \frac{1}{2} I \omega^2$$

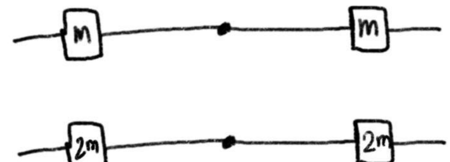
After this derivation, we see that we are left with $\sum m_i r_i^2$, which is the moment of inertia. $I = \sum m_i r_i^2$

Moment of Inertia: an object's capacity to resist rotational change; large moment of inertia = very hard to rotate

Examples:



greater moment of inertia since masses are the same but the radius is larger



greater moment of inertia because second system has a larger mass

Moment of inertia for continuous distributions of Mass:

We need to use an integral so that we can apply moment of inertia to continuous distributions of mass.

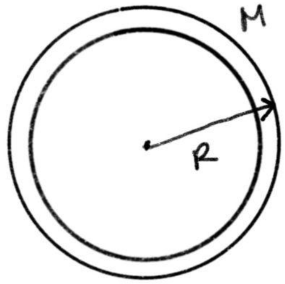
Keep in mind:

$$dm = \lambda\, dx$$
$$dm = \sigma\, dA$$
$$dm = \rho\, dV$$

Starting point for all derivations:

$$I = \sum m_i r_i^2$$
$$I = \lim_{\Delta m_i \to 0} \sum \Delta m_i r_i^2$$
$$I = \int r^2\, dm$$

Derivation for a Hoop:



$$I = \int r^2 dm$$

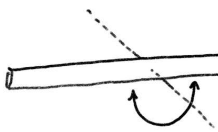
$$I = \int R^2 dm$$

R is constant

$$I = R^2 dm$$

$$I = MR^2$$

Long thin rod about the center:



$$I = \int r^2 dm$$

$$dm = \lambda dr$$

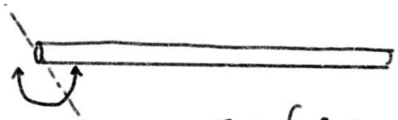
$$I = \int_{-L/2}^{L/2} r^2 \lambda dr$$

$$I = \lambda \frac{1}{3} r^3 \Big|_{-L/2}^{+L/2}$$

$$I = \frac{M}{L}\left(\frac{1}{3}\left(\frac{L}{2}\right)^3 - \frac{1}{3}\left(-\frac{L}{2}\right)^3\right)$$

$$I = \frac{1}{12} ML^2$$

Long thin rod about one end:



$$I = \int r^2 dm$$

$$dm = \lambda . dr$$

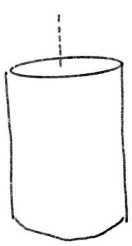
$$I = \int_0^L r^2 \lambda dr$$

$$I = \lambda \frac{1}{3} r^3 \Big|_0^L$$

$$I = \frac{M}{L}\left(\frac{L}{3}\right)^3$$

$$I = \frac{1}{3} ML^2$$

Uniform cylinder:



$$I = \int r^2 dm$$

$$dm = \rho dV$$

$$I = \int r^2 \rho dV$$

$$dV = 2\pi r L dr$$

$$I = \int r^2 \rho 2\pi r L dr$$

$$I = 2\pi L \rho \int_0^R r^3 dr$$

$$I = 2\pi L \rho \frac{1}{4} r^4 \Big|_0^R = \frac{1}{2} L\pi\rho R^4$$

$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$$

$$I = \frac{1}{2} L\pi\rho R^4 \left(\frac{M}{\pi R^2 L}\right)$$

$$I = \frac{1}{2} MR^2$$

Parallel axis theorem: $\boxed{I = I_{cm} + MD^2}$

using it w/ a long thin rod about one end

I_{cm} for middle of rod $= \frac{1}{12} ML^2$

for the end, we shift $D = \frac{1}{2}L$ so

$$I = I_{cm} + MD^2$$

$$I_{end} = \frac{1}{12} ML^2 + M\left(\frac{1}{2}L\right)^2$$

$$I_{end} = \frac{1}{12} ML^2 + \frac{1}{4} ML^2$$

$$I_{end} = \frac{1}{3} ML^2$$

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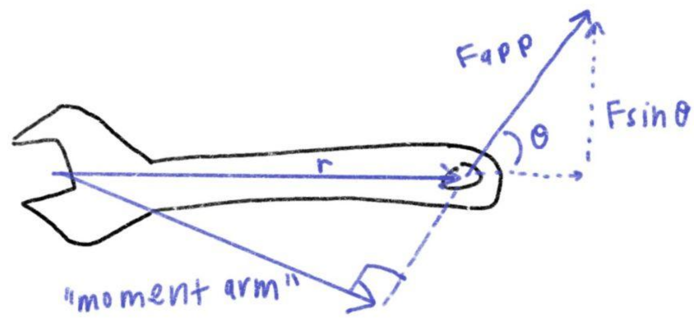
Torque and Cross Products:

There are many ways to calculate torque, the two most simple are Force multiplied by the moment arm, and Force times radius sin theta. These two methods are technically the same calculation just in different order of operations, as shown in the diagram:

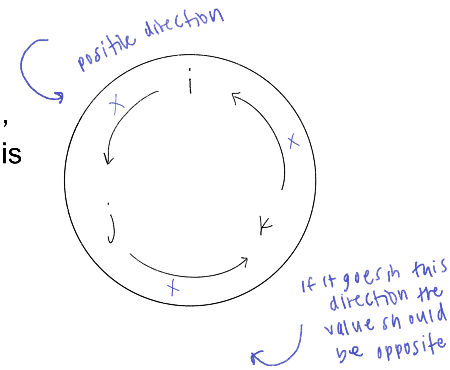


The *<u>Cross-Product</u>* *is a bit more complicated because you have to take direction into account.*
We have to first find the direction of rotation of torque using the multi-component right hand rule (not the circular right hand rule). For this analysis, the index=radius, rest of your fingers=force, and thumb=torque.

If torque is perpendicular to **r** x **f**, then when using i, j, and k, torque will be the component that is not included in the radius x force calculation. I.e. (+i) x (-j) = (-k)

$$\vec{i}\times\vec{i}=0 \qquad \vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{k}=\vec{i} \qquad \vec{k}\times\vec{i}=\vec{j}$$
$$\vec{j}\times\vec{j}=0 \qquad \vec{j}\times\vec{i}=-\vec{k} \qquad \vec{k}\times\vec{j}=-\vec{i} \qquad \vec{i}\times\vec{k}=-\vec{j}$$
$$\vec{k}\times\vec{k}=0$$

To remember these equations, you can draw this circle that tells the direction as an effect of two components:

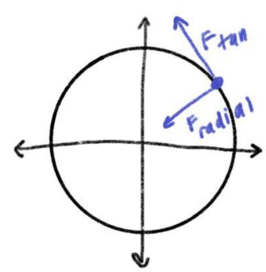


<u>Torque and angular acceleration:</u>
If you torque an object and cause it to rotate, you've applied a force and there a way to relate both the torque and the angular acceleration of the object.

$$I=\sum m_i r_i^2$$
$$I=mr^2$$

$$\tau = r\times F$$
$$\tau = rF\sin\theta$$
$$\tau = rF_{tangential}$$

$$\tau = rF_{tangential}$$
$$\tau = r(m a_{tangential})$$
$$\tau = r(m(\alpha r))$$
$$\tau = (mr^2)\alpha$$
$$\tau = I\alpha$$



Key points:
- Torque will depend on the angle at which force is being applied
- Keep in mind the order of i, j, and k when using the cross product to find torque
- Regular kinematics translates to rotational kinematics, just use omega, alpha, and theta instead of v, a, and horizontal displacement
- substitute linear/area/volume based densities into the base integral to find moments of inertia for masses with continuous distribution
- When calculating the cross product for Torque, the order of operations matters.

Important Formulae (beside kinematics)

$K_{rot}=\frac{1}{2}I\omega^2$ finding kinetic E

$I=\sum m_i r_i^2$
$I=\int r^2\,dm$
starting pts for moment of inertia

$\tau = r\times F$
$\tau = rF\sin\theta$
$\tau = Fd$
different methods to find Torque

Problems:

1. [easy] A track star runs a 400-m race on a 400-m circular track in 45 s. What is his angular velocity assuming a constant speed?
2. [medium] A baseball pitcher throws the ball in a motion where there is rotation of the forearm about the elbow joint as well as other movements. If the linear velocity of the ball relative to the elbow joint is 20.0 m/s at a distance of 0.480 m from the joint and the moment of inertia of the forearm is 0.500 kg-m^2, what is the rotational kinetic energy of the forearm?
3. [hard] A horizontal beam of length 3 m and mass 2.0 kg has a mass of 1.0 kg and width 0.2 m sitting at the end of the beam (see the following figure). What is the torque of the system about the support at the wall?

1)

400m
45s

$v = r\omega$

so $\omega = \frac{v}{r}$

$v = \frac{m}{t}$

$\omega = \frac{\left(\frac{m}{t}\right)}{r} = \frac{\left(\frac{400}{45}\right)}{\text{radius of track}}$

circumference
$400 = 2\pi r$
$200 = \pi r$
$r = 200/\pi$

$= \frac{400}{45 \cdot \left(\frac{200}{\pi}\right)}$

$= \boxed{0.140 \text{ rad/s}}$

Since this a very straightforward problem, you just have to use the question for angular velocity and plug in your values. You do however need the circumference formula to find the radius.

2) start w/ $K = \frac{1}{2} I\omega^2$

we know that

$I = 0.500 \text{ kg m}^2$

$v = 20 \text{ m/s}$

$r = 0.48 \text{m}$

first find $\omega = \frac{v}{r} = \frac{20}{0.48}$

$= 41.67 \text{ rad/s}$

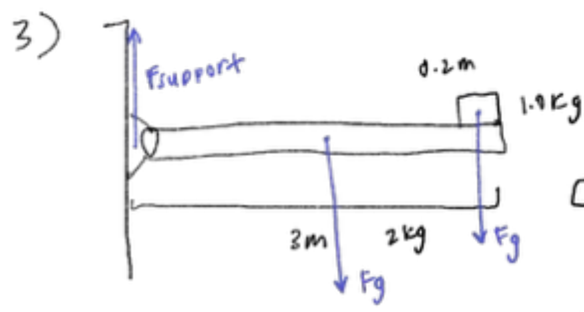
now just plug in

$K = \frac{1}{2}(0.5)(41.67)^2$

$= \boxed{434.0 \text{ J}}$

To find rotational kinetic energy, start with the equation given on the equation sheet. The problem tells us the values of moment of inertia, velocity, and radius, but to find rotational kinetic energy we need angular velocity. We find that by using the equation for angular velocity. Then plug in values.

3)



we know

$M_{beam} = 2$ beam $= 3\text{m}$

$M_{object} = 1\text{kg}$ object $= 0.2\text{m}$

start w/ $\tau = r \times F = 0$

$\tau_{support} - \tau_{beam} - \tau_{object} = 0$

$\tau_{support} = \tau_{beam} + \tau_{object}$

$\tau_{beam} = r \times F \rightarrow F_{beam} = mg = (2)(9.8)$

$\tau_{beam} = (1.5\text{m})(19.6) = 29.4 \text{ Nm}$

$\tau_{object} = r \times F \rightarrow F_{object} = mg = (1)(9.8)$

$\tau_{object} = (2.9\text{m})(9.8) = 28.42 \text{ Nm}$

$\tau_{support} = 29.4 + 28.42 = \boxed{57.82 \text{ Nm}}$

This problem is similar to other force analysis problems, just now we are applying torque. To find the torque of the system about the support, we first draw a free body diagram to figure out what forces are influencing the beam. For gravity acting on the beam, we assume it's acting on the center of mass, so the radius we would use is 1.5m. For the object at the end of the beam, the width is 0.2, but the force of gravity is also acting on the center of mass of the object. So the radius we use to calculate the torque of the object is 2.9m. Then you would just add the torque of the object to the torque of the beam.